Gaussian Mixture PHD and CPHD Filtering with Partially Uniform Target Birth

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***Abstract*—The standard Gaussian Mixture Probability Hypothesis Density (GMPHD) filter and Cardinalised Probability Hypothesis Density (GMCPHD) filter require the target birth model to take the form of a Gaussian mixture. Although any density (including a uniform density), can be approximated using a sum of Gaussians, this can be inefficient in practice, especially when a large number of Gaussians is required to achieve the desired accuracy. A better alternative in the case of an uninformative birth model would be to directly use a uniform density instead of a Gaussian mixture approximation. In this paper we present new forms of the GMPHD and GMCPHD filtering equations, which allow part of the target birth model to take on a uniform distribution, thus obviating the need to use large Gaussian mixtures to approximate a uniform birth density.**

I. Introduction

The Probability Hypothesis Density (PHD) filter [1, 2] is an algorithm for tracking multiple targets in the presence of clutter and missed detections. It was developed as an attempt to find a computationally tractable approximation to the full multi-target Bayes recursion, which is formulated using the theory of Random Finite Sets (RFS). Based on this theory, we can derive the PHD as the first statistical moment of the full multi-target probability density. The PHD is a function which characterises the intensity of targets at all points in the single-target state space, and it is therefore often referred to as the intensity function. To estimate the number of targets in a given region of the state space, we simply integrate the PHD over the region of interest. The estimated total number of targets is this integral over the entire state space, and the estimated target locations are the highest peaks of the PHD. The PHD filter propagates the intensity function in time, and corrects it based on sets of target observations. An alternative derivation of the PHD filter which does not rely on RFS theory can be found in [6].

Since the PHD filter only propagates the first moment, it discards a great deal of information about the multi-target density. To improve on this, the Cardinalised PHD (CPHD) filter was developed [2, 8]. The CPHD filter propagates the intensity function, but in addition, it also propagates a probability distribution on the number of targets, known as the cardinality distribution. This leads to the estimates of the number of targets having a lower variance.

There are two methods of implementing the (C)PHD filter, one based on a sequential Monte Carlo (SMC) representation of the intensity function [3, 4], and another based on a Gaussian mixture (GM) representation [5, 7], the second of which is the subject of this paper. The original form of the Gaussian mixture PHD (GMPHD) and Gaussian mixture CPHD (GMCPHD) filters assume that both the posterior intensity function at the previous time, and the intensity function for new targets entering the scene at the current time, are both Gaussian mixtures [5, 9]. Although it is possible to approximate any given density using a sum of Gaussians, this approach is potentially inefficient in practice since a large number of Gaussians may be necessary to obtain a reasonable approximation to the required density. This is especially true in the case where the birth density is uniform, which applies in the common situation when targets may appear anywhere with equal probability.

The use of a diffuse target birth model is addressed for the GMPHD filter in [10] and for the SMC-PHD filter in [11]. This paper adds to the work in [10] by providing a more formal derivation of the modified PHD update equation, and a direct performance comparison between the modified PHD filter and the traditional PHD filter under a GM approximation to the diffuse birth model. In addition, we extend this idea to the CPHD filter, deriving a modified form of the CPHD update equations.

The modified filter equations are derived by showing that it is possible for the PHD and CPHD filters to use a uniform birth model for the part of the target state that is directly observed through the measurements. Despite the fact that this gives a predicted intensity which is not a full Gaussian mixture, the posterior intensity after processing the measurements can still be accurately approximated by a GM. We show that this is true under the restrictions that the birth model is Gaussian for the unobserved state components, and that the uniform component of the birth density covers a large enough region of the measurement space such that the effect of truncation of the Gaussian measurements is negligible.

The main advantage of the proposed approach is that it reduces the number of parameters that must be selected by the end user. Constructing a GM birth model requires selecting a number of parameters, including the mean locations, covariance matrices and weights of the components. Poor selections can adversely affect filter performance, therefore removal of this requirement is of significant benefit. The proposed method is based on a principled approximation, making direct use of the uniform density to define the target birth model. Simulation results show that this provides better performance than a GM birth model with a small number of components, and equivalent performance to a GM birth model with a large number of components, without the computational overhead involved in using large Gaussian mixtures.

The rest of this paper is organised as follows. Section II contains the derivation of the GMPHD filter with partial uniform birth model. Section III contains the derivation for the GMCPHD with partial uniform birth model. Section IV presents some simulation results verifying the performance of these filters, and some concluding remarks are given in Section V.

II. Modified PHD Filter

The general form of the PHD filter recursion (without target spawning) is given by [1]

$$v_{k|k-1}(x) = \int P_{S,k}(\zeta) f_{k|k-1}(x|\zeta) v_{k-1}(\zeta) d\zeta + \gamma_k(x) \quad (1)$$

$$v_k(x) = (1 - P_{D,k}(x)) v_{k|k-1}(x) + \sum_{z\in Z} \frac{P_{D,k}(x) g_k(z|x) v_{k|k-1}(x)}{\kappa_k(z) + \int P_{D,k}(\xi) g_k(z|\xi) v_{k|k-1}(\xi) d\xi} \quad (2)$$

where v_{k-1} is the posterior PHD from time $k-1$, $P_{S,k}(\zeta)$ is the probability of survival at time k for a target in state ζ, $f_{k|k-1}$ is the target transition density from time $k-1$ to time k, $\gamma_k(x)$ is the prior PHD of spontaneous target births at time k, $P_{D,k}(x)$ is the probability of detection at time k for a target in state x, $g_k(z|x)$ is the likelihood function for a measurement z given target state x, and $\kappa_k(z)$ is the clutter density at time k in the vicinity of measurement z.

Let us now assume that v_{k-1} is a Gaussian mixture of the form

$$v_{k-1}(x) = \sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} \mathcal{N}(x; m_{k-1}^{(i)}, P_{k-1}^{(i)}) \quad (3)$$

Let θ represent the part of the state x that is directly observed by the measurement z, and ϕ be the part of the state which is not observed. Hence, the intensity in (3) may be expressed as

$$v_{k-1}(\theta,\phi) = \sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} \mathcal{N}(\theta,\phi; \theta_{k-1}^{(i)}, \phi_{k-1}^{(i)}, \Omega_{k-1}^{(i)}) \quad (4)$$

where $\theta_{k-1}^{(i)}$ is the observed part of $m_{k-1}^{(i)}$, $\phi_{k-1}^{(i)}$ is the unobserved part of $m_{k-1}^{(i)}$, and $\Omega_{k-1}^{(i)}$ is the covariance matrix $P_{k-1}^{(i)}$ transformed into the space defined by (θ,ϕ). In the rest of this paper, we abbreviate $\theta_k^{(i)}, \phi_k^{(i)}$ to $(\theta,\phi)_k^{(i)}$. Let us now define the intensity of spontaneous target births as

$$\gamma_k(\theta,\phi) = w_k^b U(\theta;\mathcal{B}) \mathcal{N}(\phi; \bar{\phi}, \sigma_\phi^2) \quad (5)$$

where w_k^b is the expected number of targets appearing at time k, $U(\theta;\mathcal{B})$ is the uniform distribution in θ over the region $\mathcal{B}$, $\bar{\phi}$ is the prior mean of the unmeasured state component, and σ_ϕ^2 is its prior variance. Now, the predicted PHD at time k is given by

$$v_{k|k-1}(\theta,\phi) = v_{S,k|k-1}(\theta,\phi) + \gamma_k(\theta,\phi) \quad (6)$$

where

$$v_{S,k|k-1}(\theta,\phi) = P_{S,k} \sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} \mathcal{N}(\theta,\phi; (\theta,\phi)_{k|k-1}^{(i)}, \Omega_{k|k-1}^{(i)}) \quad (7)$$

assuming the target dynamics model $f_{k|k-1}$ is linear, and the survival probability $P_{S,k}$ is independent of target state. Let us augment the state with a binary variable β so we may distinguish between the surviving components and the birth component, such that

$$v_{k|k-1}(\theta,\phi,\beta) = \begin{cases} \sum_{i=1}^{J_{k-1}} w_{k|k-1}^{(i)} \mathcal{N}(\theta,\phi; (\theta,\phi)_{k|k-1}^{(i)}, \Omega_{k|k-1}^{(i)}) & ,\ \beta = 0 \\ w_k^b U(\theta;\mathcal{B}) \mathcal{N}(\phi; \bar{\phi}, \sigma_\phi^2) & ,\ \beta = 1 \end{cases} \quad (8)$$

We now assume that new targets are always detected, and for simplicity we also assume that the detection probability for surviving targets is independent of target state. This leads to the definition

$$P_{D,k}(\theta,\phi,\beta) = \begin{cases} P_{D,k} & ,\ \beta = 0 \\ 1 & ,\ \beta = 1 \end{cases} \quad (9)$$

Let us also assume that the measurement likelihood function is Gaussian and defined by

$$g(z|\theta,\phi,\beta) = \mathcal{N}(z; \theta, \sigma_\theta^2) \quad (10)$$

The posterior intensity is now given by (11) and (12), where, for the sake of brevity, the subscript $k|k-1$ is omitted for $w^{(i)}$, $\theta^{(i)}$, $(\theta,\phi)^{(i)}$ and $\Omega^{(i)}$.

$$v_k(\theta,\phi,0) = (1 - P_{D,k}) \sum_{i=1}^{J_{k-1}} w^{(i)} \mathcal{N}(\theta,\phi; (\theta,\phi)^{(i)}, \Omega^{(i)}) + \sum_{z\in Z} \sum_{i=1}^{J_{k-1}} \frac{P_{D,k} \mathcal{N}(z; \theta^{(i)}, \sigma_\theta^2) w^{(i)} \mathcal{N}(\theta,\phi; (\theta,\phi)^{(i)}, \Omega^{(i)})}{\kappa_k(z) + \int P_{D,k}(\xi) g(z|\xi) v_{k|k-1}(\xi) d\xi} \quad (11)$$

$$v_k(\theta,\phi,1) = \sum_{z\in Z} \frac{\mathcal{N}(\theta; z, \sigma_\theta^2) w_k^b U(\theta;\mathcal{B}) \mathcal{N}(\phi; \bar{\phi}, \sigma_\phi^2)}{\kappa_k(z) + \int P_{D,k}(\xi) g(z|\xi) v_{k|k-1}(\xi) d\xi} \quad (12)$$

On the denominator in (11) and (12) we have

$$\begin{aligned}
&\int P_{D,k}(\xi)g(z|\xi)v_{k|k-1}(\xi)d\xi \\
&\quad= \int\int\int P_{D,k}(\theta,\phi,\beta)g(z|\theta,\phi,\beta)v_{k|k-1}(\theta,\phi,\beta)d\theta d\phi d\beta \\
&\quad= \int\int\sum_{\beta=0}^{1} P_{D,k}(\theta,\phi,\beta)g(z|\theta,\phi,\beta)v_{k|k-1}(\theta,\phi,\beta)d\theta d\phi \\
&\quad= \int\int P_{D,k}\sum_{i=1}^{J_{k-1}} w_{k|k-1}^{(i)}\mathcal{N}(z;\theta_{k|k-1}^{(i)},\sigma_\theta^2) \\
&\qquad\qquad \mathcal{N}(\theta,\phi;(\theta,\phi)_{k|k-1}^{(i)},\Omega_{k|k-1}^{(i)})d\theta d\phi \\
&\qquad + w_k^b\int\int\mathcal{N}(\theta;z,\sigma_\theta^2)\mathcal{N}(\phi;\bar{\phi},\sigma_\phi^2)U(\theta;\mathcal{B})d\theta d\phi \\
&\quad= \sum_{i=1}^{J_{k-1}} P_{D,k}w_{k|k-1}^{(i)}q_k^{(i)}(z) \\
&\qquad\qquad \int\int\mathcal{N}(\theta,\phi;(\theta,\phi)_k^{(i)},\Omega_k^{(i)})d\theta d\phi \\
&\qquad + w_k^b\int\mathcal{N}(\phi;\bar{\phi},\sigma_\phi^2)d\phi\int\mathcal{N}(\theta;z,\sigma_\theta^2)\frac{\iota(\mathcal{B})}{V(\mathcal{B})}d\theta
\end{aligned} \tag{13}$$

where $q_k^{(i)}(z) = \mathcal{N}(z;\theta_{k|k-1}^{(i)},S_{k|k-1}^{(i)})$ is the likelihood of measurement z against component i (with $S_{k|k-1}^{(i)}$ being the innovation covariance for component i), $\iota(\mathcal{B})$ is the indicator function for the region $\mathcal{B}$, and $V(\mathcal{B})$ is the volume of region $\mathcal{B}$. We now make the following approximation, which is justified as long as most of the probability mass of $\mathcal{N}(\theta;z,\sigma_\theta^2)$ is contained in the region $\mathcal{B}$.

$$\int\mathcal{N}(\theta;z,\sigma_\theta^2)\frac{\iota(\mathcal{B})}{V(\mathcal{B})}d\theta \approx \frac{1}{V(\mathcal{B})} \tag{14}$$

This results in the following approximation for the denominator

$$\begin{aligned}
&\int P_{D,k}(\xi)g(z|\xi)v_{k|k-1}(\xi)d\xi \\
&\qquad\approx \kappa_k(z) + P_{D,k}\sum_{i=1}^{J_{k-1}} w_{k|k-1}^{(i)}q_k^{(i)}(z) + w_k^b/V(\mathcal{B})
\end{aligned} \tag{15}$$

Substituting (15) into (11) and (12) gives us the closed form of the posterior PHD

$$\begin{aligned}
v_k(\theta,\phi,0) &= \sum_{i=1}^{J_{k-1}} w_{m,k}^{(i)}\mathcal{N}(\theta,\phi;(\theta,\phi)_{k|k-1}^{(i)},\Omega_{k|k-1}^{(i)}) \\
&\quad + \sum_{z\in Z}\sum_{i=1}^{J_{k-1}} w_{s,k}^{(i)}\mathcal{N}(\theta,\phi;(\theta,\phi)_k^{(i)},\Omega_k^{(i)})
\end{aligned} \tag{16}$$

$$v_k(\theta,\phi,1) = \sum_{z\in Z} w_{b,k}^{(i)}\mathcal{N}(\theta;z,\sigma_\theta^2)\mathcal{N}(\phi;\bar{\phi},\sigma_\phi^2) \tag{17}$$

where

$$w_{m,k}^{(i)} = (1-P_{D,k})w_{k|k-1}^{(i)} \tag{18}$$

$$w_{s,k}^{(i)} = \frac{P_{D,k}q_k^{(i)}(z)w_{k|k-1}^{(i)}}{\kappa_k(z)+P_{D,k}\sum_{i=1}^{J_{k-1}} w_{k|k-1}^{(i)}q_k^{(i)}(z)+w_k^b/V(\mathcal{B})} \tag{19}$$

$$w_{b,k}^{(i)} = \frac{w_k^b/V(\mathcal{B})}{\kappa_k(z)+P_{D,k}\sum_{i=1}^{J_{k-1}} w_{k|k-1}^{(i)}q_k^{(i)}(z)+w_k^b/V(\mathcal{B})} \tag{20}$$

Since newborn targets at time k become surviving targets at time $k+1$, the predicted PHD for surviving targets at time $k+1$ is obtained by propagating the sum of both the $\beta=0$ and $\beta=1$ components of the posterior PHD at time k.

III. Modified CPHD Filter

In this section we derive the CPHD equations for the case in which the target birth model is uniform on the measured part of the state space. Assume that the posterior PHD at time $k-1$ is a Gaussian mixture of the same form as in (3). Let ρ_{k-1} denote the posterior cardinality distribution at time $k-1$, and $\rho_{\Gamma,k}$ denote the prior cardinality distribution of spontaneous births at time k. Then, as per the standard GMCPHD filter, the predicted cardinality distribution at time k is given by the convolution [8]

$$\rho_{k|k-1}(n) = \sum_{j=0}^{n}\rho_{\Gamma,k}(n-j)\sum_{l=j}^{\infty}C_j^l\rho_{k-1}(l)P_{S,k}^j(1-P_{S,k})^{l-j} \tag{21}$$

where $C_j^l = \frac{l!}{j!(l-j)!}$ is the binomial coefficient. The target birth intensity is the same as defined in (5), and the predicted intensity function at time $k-1$ is the same as (6). As we did for the PHD filter, we again augment the state with the variable β to distinguish surviving components from the birth component, resulting in the definition (8) for the predicted intensity, and (9) for the detection probability. Using θ and ϕ to separate the measured and unmeasured components of the state, and making the following definitions to simplify the notation,

$$\chi = \frac{\langle\Upsilon_k^1[v_{k|k-1},Z_k],\rho_{k|k-1}\rangle}{\langle\Upsilon_k^0[v_{k|k-1},Z_k],\rho_{k|k-1}\rangle} \tag{22}$$

$$\chi(z) = \frac{\langle\Upsilon_k^1[v_{k|k-1},Z_k-\{z\}],\rho_{k|k-1}\rangle}{\langle\Upsilon_k^0[v_{k|k-1},Z_k],\rho_{k|k-1}\rangle} \tag{23}$$

where $\langle\cdot,\cdot\rangle$ is the inner product between two functions, the CPHD update can be expressed as

$$\rho_k(n) = \frac{\Upsilon_k^0[v_{k|k-1},Z_k](n)\rho_{k|k-1}(n)}{\langle\Upsilon_k^0[v_{k|k-1},Z_k],\rho_{k|k-1}\rangle} \tag{24}$$

$$\begin{aligned}
v_k(\theta,\phi,0) &= (1-P_{D,k})\chi v_{k|k-1}(\theta,\phi,0) \\
&\quad + \sum_{z\in Z}\psi_{k,z}(\theta,\phi,0)\chi(z)v_{k|k-1}(\theta,\phi,0)
\end{aligned} \tag{25}$$

$$v_k(\theta,\phi,1) = \sum_{z\in Z}\psi_{k,z}(\theta,\phi,1)\chi(z)v_{k|k-1}(\theta,\phi,1) \tag{26}$$

where

$$\Upsilon_k^u[v,Z](n) = \sum_{j=0}^{\min(|Z|,n)} (|Z|-j)! P_{K,k}(|Z|-j) P_{j+u}^n \frac{\langle 1-P_{D,k}, v\rangle^{n-(j+u)}}{\langle 1, v\rangle^n} e_j(\Xi_k(v,Z)) \tag{27}$$

$$\psi_{k,z}(\theta,\phi,\beta) = \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} g(z|\theta,\phi) P_{D,k}(\theta,\phi,\beta) \tag{28}$$

$$\Xi_k(v,Z) = \{\langle v, \psi_{k,z}\rangle : z \in Z\} \tag{29}$$

$$P_{j+u}^n = \frac{n!}{(n-(j+u))!} \tag{30}$$

$P_{K,k}(\cdot)$ is the cardinality distribution of clutter at time k, and $e_j(\cdot)$ is the elementary symmetric function of order j. Now, the inner product between $v_{k|k-1}$ and $\psi_{k,z}$ is given by

$$\begin{aligned}
\langle v_{k|k-1}, \psi_{k,z}\rangle &= \int\int\int v_{k|k-1}(\theta,\phi,\beta)\psi_{k,z}(\theta,\phi,\beta) d\theta d\phi d\beta \\
&= \int\int \sum_{\beta=0}^{1} v_{k|k-1}(\theta,\phi,\beta)\psi_{k,z}(\theta,\phi,\beta) d\theta d\phi \\
&= \int\int \Bigg[\sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} \mathcal{N}(\theta,\phi;(\theta,\phi)_{k|k-1}^{(i)}, \Omega_{k|k-1}^{(i)}) \\
&\qquad \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} \mathcal{N}(z;\theta,\sigma_\theta^2) P_{D,k}(r,\theta,0) \Bigg] \\
&\qquad + w_k^b \mathcal{N}(\phi;\bar{\phi},\sigma_\phi^2) U(\theta;\mathcal{B}) \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} \mathcal{N}(z;\theta,\sigma_\theta^2) \\
&\qquad P_{D,k}(\theta,\phi,1) d\theta d\phi \\
&= \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} P_{D,k} \sum_{i=1}^{J_{k|k-1}} \int\int w_{k|k-1}^{(i)} q_k^{(i)}(z) \\
&\qquad \mathcal{N}(\theta,\phi;(\theta,\phi)_k^{(i)}, \Omega_k^{(i)}) d\phi d\theta \\
&\quad + \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} w_k^b \int \mathcal{N}(\phi;\bar{\phi},\sigma_\phi^2) d\phi \\
&\qquad \int \mathcal{N}(\theta;z,\sigma_\theta^2) \frac{\iota(\mathcal{B})}{V(\mathcal{B})} d\theta \\
&\approx \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} \left(\frac{w_k^b}{V(\mathcal{B})} + P_{D,k} \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} q_k^{(i)}(z) \right)
\end{aligned} \tag{31}$$

Hence, the elementary symmetric functions are calculated over the set defined by

$$\Xi_k(v_{k|k-1}, Z) = \left\{ \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} \left(\frac{w_k^b}{V(\mathcal{B})} + P_{D,k} \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} q_k^{(i)}(z) \right) : z \in Z \right\}$$

The remaining inner products required in the CPHD calculations are

$$\begin{aligned}
&\langle 1, v_{k|k-1}\rangle \\
&= \int\int \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} \mathcal{N}(\theta,\phi;(\theta,\phi)_{k|k-1}^{(i)}, \Omega_{k|k-1}^{(i)}) \\
&\qquad + w_k^b \mathcal{N}(\phi;\bar{\phi},\sigma_\phi^2) U(\theta;\mathcal{B}) d\theta d\phi \\
&= \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} + w_k^b \int \mathcal{N}(\phi;\bar{\phi},\sigma_\phi^2) d\phi \int U(\theta;\mathcal{B}) d\theta \\
&= \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} + w_k^b \\
&= \langle 1, w_{k|k-1}\rangle + w_k^b
\end{aligned} \tag{32}$$

and

$$\langle 1-P_{D,k}, v_{k|k-1}\rangle = \sum_{i=1}^{J_{k|k-1}} (1-P_{D,k}) w_{k|k-1}^{(i)} \tag{33}$$

$$= (1-P_{D,k})\langle 1, w_{k|k-1}\rangle \tag{34}$$

which results in the following equation for Υ_k^u

$$\begin{aligned}
&\Upsilon_k^u[v_{k|k-1}, Z](n) \\
&= \sum_{j=0}^{\min(|Z|,n)} (|Z|-j)! P_{K,k}(|Z|-j)(1-P_D)^{n-(j+u)} \\
&\qquad \frac{\langle 1, w_{k|k-1}\rangle^{n-(j+u)}}{\left(\langle 1, w_{k|k-1}\rangle + w_k^b\right)^n} P_{j+u}^n e_j\left(\Xi_k(v_{k|k-1}, Z)\right)
\end{aligned} \tag{35}$$

Now, the closed form of the posterior PHD is given by

$$\begin{aligned}
v_k(\theta,\phi,0) &= (1-P_{D,k})\chi v_{k|k-1}(\theta,\phi,0) \\
&\qquad + \sum_{z\in Z} \psi_{k,z}(\theta,\phi,0)\chi(z) v_{k|k-1}(\theta,\phi,0) \\
&= (1-P_{D,k})\chi \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} \mathcal{N}(\theta,\phi;(\theta,\phi)_{k|k-1}^{(i)}, \Omega_{k|k-1}^{(i)}) \\
&\quad + \sum_{z\in Z} \sum_{i=1}^{J_{k|k-1}} P_{D,k} \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} \chi(z) w_{k|k-1}^{(i)} \mathcal{N}(\theta;z,\sigma_\theta^2) \\
&\qquad \mathcal{N}(\theta,\phi;(\theta,\phi)_{k|k-1}^{(i)}, \Omega_{k|k-1}^{(i)}) \\
&= (1-P_{D,k})\chi \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} \mathcal{N}(\theta,\phi;(\theta,\phi)_{k|k-1}^{(i)}, \Omega_{k|k-1}^{(i)}) \\
&\quad + \sum_{z\in Z} \sum_{i=1}^{J_{k|k-1}} P_{D,k} \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} \chi(z) w_{k|k-1}^{(i)} q_k^{(i)}(z) \\
&\qquad \mathcal{N}(\theta,\phi;(\theta,\phi)_k^{(i)}, \Omega_k^{(i)})
\end{aligned}$$

$$= \sum_{i=1}^{J_{k|k-1}} w_{k,m}^{(i)} \mathcal{N}(\theta, \phi; (\theta,\phi)_{k|k-1}^{(i)}, \Omega_{k|k-1}^{(i)}) + \sum_{z\in Z} \sum_{i=1}^{J_{k|k-1}} w_{k,s}^{(i)}(z) \mathcal{N}(\theta, \phi; (\theta,\phi)_{k}^{(i)}, \Omega_{k}^{(i)}) \quad (36)$$

and

$$\begin{aligned} v_k(\theta,\phi,1) &= \sum_{z\in Z} \psi_{k,z}(\theta,\phi,1)\chi(z) v_{k|k-1}(\theta,\phi,1) \\ &= \sum_{z\in Z} \chi(z) \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} \mathcal{N}(\theta; z, \sigma_\theta^2) v_{k|k-1}(\theta,\phi,1) \\ &= \sum_{z\in Z} \chi(z) \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} w_k^b \mathcal{N}(\theta; z, \sigma_\theta^2) \mathcal{N}(\phi; \bar{\phi}, \sigma_\phi^2) U(\theta; \mathcal{B}) \\ &= \sum_{z\in Z} \chi(z) \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} w_k^b \mathcal{N}(\theta; z, \sigma_\theta^2) \mathcal{N}(\phi; \bar{\phi}, \sigma_\phi^2) \frac{1}{V(\mathcal{B})} \\ &= \sum_{z\in Z} w_{k,b}(z) \mathcal{N}(\theta; z, \sigma_\theta^2) \mathcal{N}(\phi; \bar{\phi}, \sigma_\phi^2) \end{aligned} \quad (37)$$

where

$$w_{k,m}^{(i)} = (1 - P_{D,k}) w_{k|k-1}^{(i)} \chi \quad (38)$$

$$w_{k,s}^{(i)}(z) = \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} P_{D,k} w_{k|k-1}^{(i)} q_k^{(i)}(z) \chi(z) \quad (39)$$

$$w_{k,b}(z) = \frac{\langle 1,\kappa_k\rangle}{\kappa_k(z)} \frac{w_k^b}{V(\mathcal{B})} \chi(z) \quad (40)$$

Thus, the posterior intensity function is a Gaussian mixture as required. As for the PHD filter, the $\beta = 0$ and $\beta = 1$ components of the posterior intensity at time k are added together in the prediction at time $k+1$.

The calculation of the posterior cardinality distribution remains unchanged from the original GMCPHD filter, except for the new definition of $\Upsilon_k^u[v_{k|k-1}, Z](n)$. Hence, the posterior cardinality distribution is given by [8]

$$\rho_{k|k}(n) = \frac{\Upsilon_k^0[v_{k|k-1}, Z_k](n) \rho_{k|k-1}(n)}{\langle \Upsilon_k^0[v_{k|k-1}, Z_k], \rho_{k|k-1}\rangle} \quad (41)$$

which completes the derivation of the modified GMCPHD filter.

IV. Simulation Results

In this section we present some simulation results comparing the performance of the PHD and CPHD filters with uniform birth model, against the equivalent filter using a traditional Gaussian mixture birth model. The test problem is a simple linear multi-target tracking scenario, consisting of up to six nearly constant velocity targets within a square surveillance region of size 15km. Four of the targets are present at the beginning, with two more appearing after 25 scans, and one disappearing after 80 scans.

The target state is a 4-element vector consisting of the Cartesian position and velocity. The sensor measures the position elements of the state directly, therefore we use the standard linear Kalman filter to update the components of the PHD. The sampling period of the sensor is 20 seconds, and the simulation runs for a total of 100 scans. The clutter measurements are uniformly distributed throughout the surveillance region, with an average intensity of 100 per scan. The standard deviation of the measurement noise is 100m, and the detection probability for all targets is 0.95. For all filters, the birth intensity is set to 0.05 new targets per scan, the Malahanobis distance threshold for component merging is set to 4, and the weight threshold for elimination of mixture components is 10^{-5}. The scenario geometry, and an example measurement set is shown in Figure 1. All performance metrics quoted are averaged over 300 independent Monte Carlo runs of this scenario.

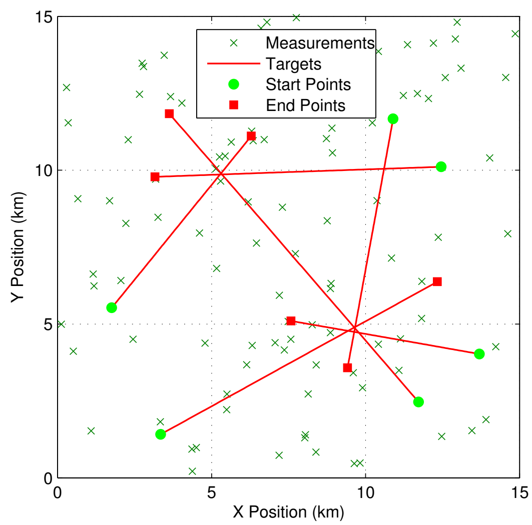

Fig. 1: Test scenario

The results of both filters under the uniform birth model are compared with Gaussian mixture (GM) birth models with different numbers of components. To construct the GM birth models, we position the means of the Gaussians evenly on a grid covering the surveillance region, each component having an equal covariance large enough such that the entire region is adequately covered by the mixture. As the number of components in the model increases, we use a smaller covariance since the spacing between Gaussians is reduced. Figure 2 shows the birth model using a 2×2 grid (GM2), a 5×5 grid (GM5), and a 10×10 grid (GM10), where the circles represent the 90% confidence region of each component, and the square shows the surveillance region.

We begin with the results of the GMPHD filter, for which we compare the uniform birth model with the GM2, GM5 and GM10 birth models, in addition to a more extreme 50×50 (GM50) model. Figures 3 and 4 show the optimal subpattern assignment (OSPA) metric [12] for all five cases, and the algorithm execution time is shown in Figure 5.

As one would expect, we find that the GM2 birth model performs poorly compared to the others, since the diffuse Gaussians result in increased localisation error. The GM5 and GM10 models perform better, although the error is still slightly

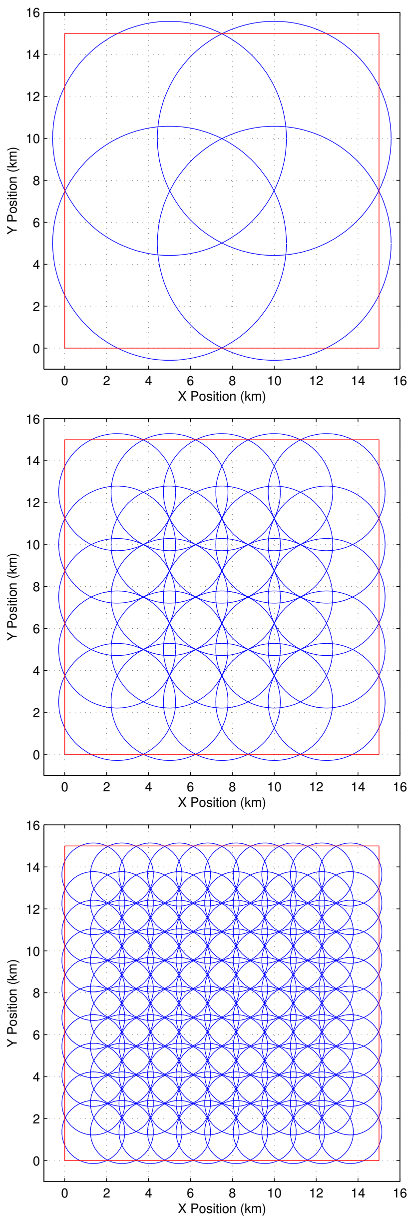

Fig. 2: GM2, GM5 and GM10 birth models

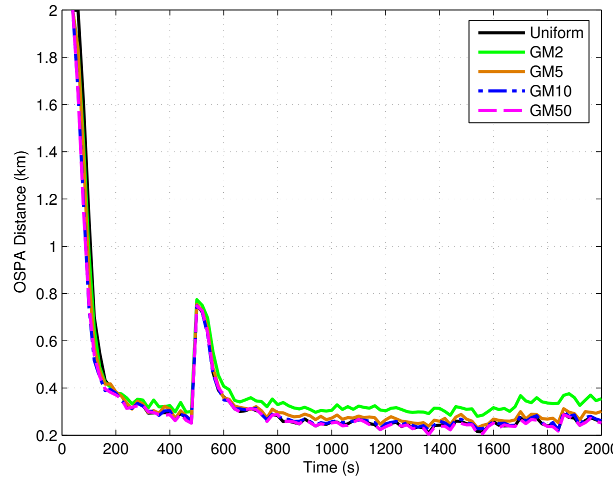

Fig. 3: OSPA for GMPHD filters

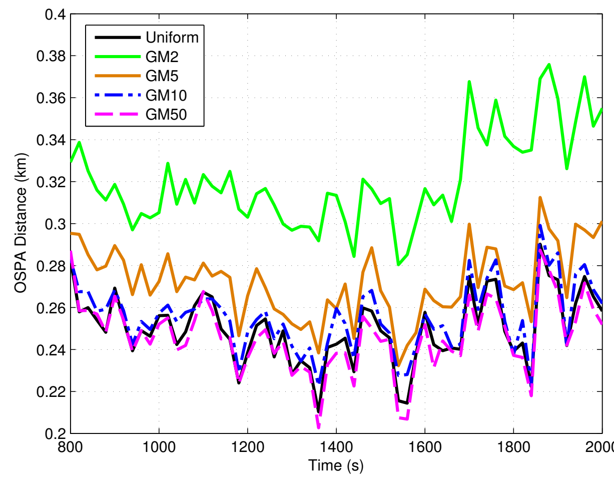

Fig. 4: OSPA for GMPHD filters (zoomed)

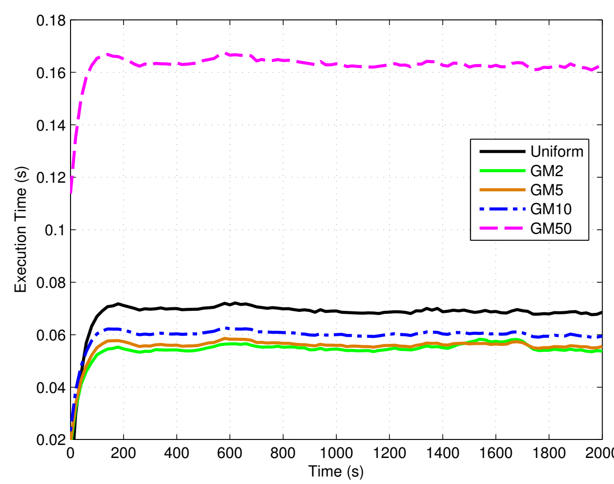

Fig. 5: Execution time for GMPHD filters

higher than that of the GM50 and uniform models, which have very similar performance. This behaviour suggests that the GM birth model becomes almost equivalent to the proposed uniform birth model as the number of components increases. This is an intuitively appealing result, since the Gaussian mixture does indeed converge to a uniform distribution as the number of components becomes larger. The execution time when using the GM50 model was greater compared to the uniform model due to the large number of birth components.

The GMCPHD filter exhibits similar characteristics to the GMPHD. Figures 6 and 7 show the OSPA for the five different birth models, and the execution time is shown in Figure 8. Again, we find that the GM2 model performs poorly in comparison to the others since it provides a less accurate

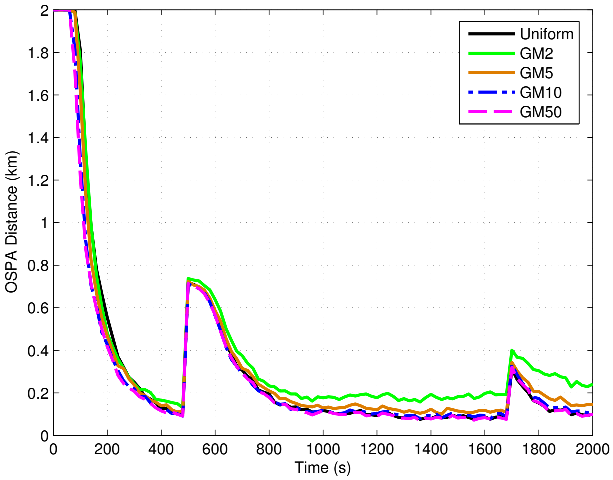

Fig. 6: OSPA for GMCPHD filters

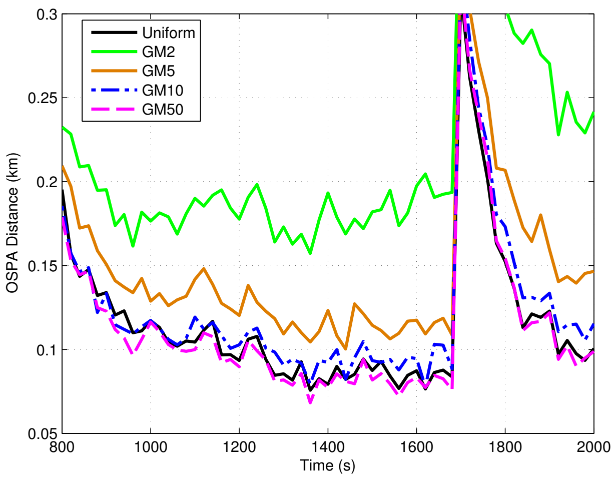

Fig. 7: OSPA for GMCPHD filters (zoomed)

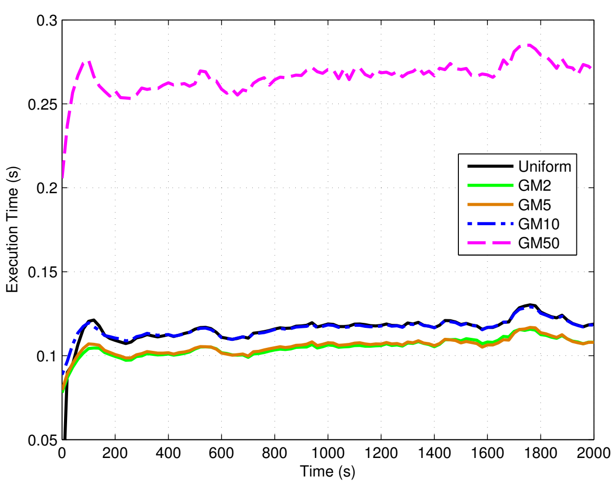

Fig. 8: Execution time for GMCPHD filters

approximation to the uniform distribution. The performance improves as the number of components increases, but the performance never significantly exceeds that of the uniform birth model, and the execution time increases when a very large number of components are used.

V. Conclusion

In this paper we have derived new forms of the GMPHD and GMCPHD filters which allow a uniform distribution to be used in the prior target birth model for the part of the state space which is directly observed through the measurements. Although this leads to a predicted intensity function which is not a Gaussian mixture, if the measurements are Gaussian, it is possible to obtain an accurate Gaussian mixture approximation to the posterior intensity. The modified GMPHD and GMCPHD filters were tested by Monte Carlo simulations on a simple linear-Gaussian scenario. The results show that the performance when using the uniform birth model is very close to that of a GM birth model with enough components to adequately approximate the uniform distribution. The advantage of the uniform approach is that it provides very similar performance to a large GM birth model, while avoiding both the need to tune many birth model parameters, and the computational overhead involved with using a large Gaussian mixture.

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