Multi-target Filtering with Unknown Clutter Density using a Bootstrap GMCPHD Filter

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Abstract—**It was recently demonstrated that the Gaussian Mixture Cardinalised Probability Hypothesis Density (GMCPHD) filter can be used when the clutter density is unknown. Here we examine the performance of this filter, and as one would expect, it does not do as well as the conventional GMCPHD with matched clutter density. To improve the performance, we propose a bootstrap filtering scheme, and demonstrate by simulations on a bearings-only multi-target filtering scenario, that it is capable of performing almost as well as the matched GMCPHD filter.**

Index Terms—**Adaptive filtering, clutter rate estimation, multi-target filtering.**

I. INTRODUCTION

Multi-target filtering in the presence of clutter is an important problem which has undergone a great deal of research, however it is often assumed that the clutter density is a known parameter which can be provided as prior information. This is unrealistic for real-world applications, as it is often impossible to know this parameter accurately ahead of time. Furthermore, its value is often dependent on environmental conditions which may vary with time, leading to the requirement for operator intervention to maintain reasonable performance. It is therefore important to investigate trackers which do not require the clutter density to be specified, and can instead estimate it on-line.

While the original CPHD filter [1], [2] requires knowledge of the clutter density, it was demonstrated in [3] that this filter can also be used to jointly estimate the clutter density and target states. This allows filtering in the presence of an unknown and time-varying clutter background, a technique we shall refer to as λ-CPHD filtering. This method uses the concept of 'clutter targets' to explain the presence of clutter measurements. Just like real targets, clutter targets are modelled with a detection probability, survival probability, and birth rate. However, since the cardinality distribution now includes both real and clutter targets, there is no way to obtain the cardinality distribution for real targets only using this method. For this reason, the performance of λ-CPHD is significantly worse than that of the conventional CPHD if the clutter density is accurately known. To improve upon this we propose a simple bootstrap filtering scheme in which we use the λ-CPHD filter to estimate the clutter density only. This is subsequently passed to a conventional CPHD filter which estimates the PHD and cardinality distribution for real targets.

Some traditional techniques for tracking with unknown clutter density include non-parametric versions of Probabilistic Data Association (PDA) [4] and Integrated PDA (IPDA) [5] for the case of a single target in clutter, and non-parametric Joint PDA (JPDA) [6] and Joint Integrated PDA (JIPDA) [7] for multiple targets in clutter. The drawback of JPDA and JIPDA is that their computational complexity increases exponentially as the target density increases, making them unsuitable in many situations. Some other independent techniques for on-line clutter rate estimation have also been proposed in [8] and [9], and comparison of these with the bootstrap GMCPHD may be the subject of future work.

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II. PROBLEM FORMULATION

This letter addresses the general problem of multi-target filtering against a clutter background with unknown density. The proposed technique can be applied to any type of sensor or target model, but for the purposes of illustration, we consider the application to bearings-only filtering in this letter.

Consider a single passive sensor on board a manoeuvring platform, whose position at time index k is denoted by the Cartesian coordinates (x_k^s, y_k^s), which is known at all times. Multiple targets may be present in the sensor's detection region, and their dynamics are assumed to follow a nearly constant velocity model, defined as follows. Consider a target t, which at time index k has Cartesian position coordinates (x_k^t, y_k^t) and instantaneous velocity vector $(\dot{x}_k^t, \dot{y}_k^t)$. The state vector for target t at time k is defined as $\boldsymbol{x}_{k,t} = [x_k^t, y_k^t, \dot{x}_k^t, \dot{y}_k^t]^T$ and the state transition model is

$$\begin{aligned} \boldsymbol{x}_{k+1,t} &= \boldsymbol{F}\boldsymbol{x}_{k,t} + \boldsymbol{\Gamma}\boldsymbol{v}_k \\ \boldsymbol{F} &= \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \otimes \boldsymbol{I}_2, \quad \boldsymbol{\Gamma} = \begin{bmatrix} T^2/2 \\ T \end{bmatrix} \otimes \boldsymbol{I}_2, \end{aligned} \tag{1}$$

where T is the measurement scan interval, and $\boldsymbol{v}_k \sim \mathcal{N}(0, \boldsymbol{Q})$ is a 2×1 i.i.d Gaussian process noise vector with $\boldsymbol{Q} = \sigma_v^2 \boldsymbol{I}_2$, where σ_v is the standard deviation of the process noise acceleration. The sensor measures target t bearing according to $z_k^t = h(\boldsymbol{x}_{k,t}, \boldsymbol{x}_{k,s}) + w_k$, where

$$h(\boldsymbol{x}_{k,t}, \boldsymbol{x}_{k,s}) = \arctan\left(\frac{x_k^t - x_k^s}{y_k^t - y_k^s}\right) \tag{2}$$

$w_k \sim \mathcal{N}(0, \sigma^2)$ is zero-mean white Gaussian measurement noise with standard deviation σ, and $\boldsymbol{x}_{k,s}$ denotes the sensor state vector. For each target t in X_k, the sensor generates a Bernoulli measurement random finite set (RFS) denoted $D_k(\boldsymbol{x}_{k,t})$, with an existence parameter $P_{D,k}$ (the detection probability), and likelihood function $g(z_k^t | \boldsymbol{x}_{k,t}, \boldsymbol{x}_{k,s}) = \mathcal{N}(z_k^t; h(\boldsymbol{x}_{k,t}, \boldsymbol{x}_{k,s}), \sigma_\theta^2)$. Thus the RFS of true target detections at k is given by $\Theta_k(X_k) = \bigcup_{\boldsymbol{x}_{k,t} \in X_k} D_k(\boldsymbol{x}_{k,t})$. At each time k, the sensor also produces a Poisson RFS of false measurements denoted K_k, the intensity of which is unknown to the filter and may vary with time. The overall set of measurements at time k can thus be expressed as $Z_k = \Theta_k(X_k) \cup K_k$. The goal is to recursively estimate the true target sets X_k at each k, conditioned on all measurement sets received up to k.

III. GMCPHD Filter for Unknown Clutter Rates

In [3], the λ-GMCPHD filter was presented as a method of dealing with cases in which the clutter rate is unknown and possibly time varying. As with the original GMCPHD filter [2], the λ-GMCPHD requires the use of a Gaussian mixture birth model, which can be restrictive when the prior distribution for new targets is in fact uniform. An alternative implementation of GMCPHD allowing the birth model to be uniformly distributed on the measurement space is derived in [10], and in what follows we have applied the same principle to the λ-GMCPHD filter.

We begin by defining some notation. Let $\tilde{v}_k(\cdot)$ denote the posterior PHD of a composite multi-target state comprising both real targets and clutter targets, and let $\tilde{\rho}_k(\cdot)$ denote the cardinality distribution of this state. The composite PHD $\tilde{v}_k(\cdot)$ can be decomposed as

$$\tilde{v}_k(\tilde{x}) = \begin{cases} v_k^{(1)}(x) & \text{if } \tilde{x} = x \\ v_k^{(0)}(c) & \text{if } \tilde{x} = c \end{cases} \tag{3}$$

where $v_k^{(1)}(\cdot)$ is the PHD for real targets, and $v_k^{(0)}$ is the PHD for clutter. As in the conventional GMCPHD filter, we represent $v_k^{(1)}(\cdot)$ using a Gaussian mixture, and the probabilities of detection and survival for real targets are denoted $p_{D,k}^{(1)}$ and $p_{S,k}^{(1)}$ respectively. The PHD for clutter targets $v_k^{(0)}(\cdot)$ is completely characterised by the number of clutter targets, which we denote as $N_k^{(0)}$.

Clutter is modelled in terms of the following parameters; the mean number of clutter target births per frame $N_{\Gamma,k}^{(0)}$, the probability of clutter target survival $p_{S,k}^{(0)}$, the probability of clutter target detection $p_{D,k}^{(0)}$, and the spatial likelihood $\alpha_k(z)$. The choice of values for these parameters is governed by the desired filter response. Lower values of $p_{D,k}^{(0)}$ lead to reduced variance in the clutter rate estimates, at the expense of a longer response time to changes in the true clutter rate. It can also be shown that if the true clutter rate is constant, the filter will give the most accurate estimates when that rate is $(N_{\Gamma,k}^{(0)} p_{D,k}^{(0)})/(1 - p_{S,k}^{(0)})$.

A. Prediction

At time $k-1$, let $\tilde{\rho}_{k-1}$ denote the posterior cardinality distribution, $v_{k-1}^{(1)}$ the posterior PHD of the form

$$v_{k-1}^{(1)}(x) = \sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} \mathcal{N}(x; m_{k-1}^{(i)}, P_{k-1}^{(i)}). \tag{4}$$

in which $w_{k-1}^{(i)}$ is the weight of the i-th Gaussian component, and $N_{k-1}^{(0)}$ the posterior number of clutter generators. Let (θ, r, c, s) represent the state space in terms of bearing, range, course and speed, and let $\Psi(x;\gamma)$ represent a function which transforms the density γ into the Cartesian space [11]. Then the predicted PHD at time k is given by

$$v_{k|k-1}(x) = v_{S,k|k-1}(x) + \Psi(x; \gamma_k(\theta, r, c, s)) \tag{5}$$

where,

$$v_{S,k|k-1}(x) = p_{S,k}^{(1)} \sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} \mathcal{N}\left(x; m_{k|k-1}^{(i)}, P_{k|k-1}^{(i)}\right) \tag{6}$$

$$\gamma_k(\theta, r, c, s) = w_k^b U(\theta; [0, 2\pi]) \mathcal{N}(r; \bar{r}, \sigma_r^2) \times \mathcal{N}(c; \theta - \pi, \sigma_c^2) \mathcal{N}(s; \bar{s}, \sigma_s^2) \tag{7}$$

$$m_{k|k-1}^{(i)} = \boldsymbol{F} m_{k-1}^{(i)} \tag{8}$$

$$P_{k|k-1}^{(i)} = \boldsymbol{F} P_{k-1}^{(i)} \boldsymbol{F}^T + \boldsymbol{\Gamma} \boldsymbol{Q} \boldsymbol{\Gamma}^T \tag{9}$$

$\bar{r}$ and σ_r^2 are the prior range and range variance for new targets, $\bar{s}$ and σ_s^2 are the prior speed and speed variance for new targets, σ_c^2 is the prior course variance for new targets, and $U(\theta; [0, 2\pi])$ is the pdf of the uniform distribution over the interval $[0, 2\pi]$. The predictions for the number of clutter generators, clutter rate, and cardinality distribution are

$$N_{k|k-1}^{(0)} = N_{\Gamma,k}^{(0)} + p_{S,k}^{(0)} N_{k-1}^{(0)} \tag{10}$$

$$\lambda_{k|k-1}^{(0)} = N_{k|k-1}^{(0)} p_{D,k}^{(0)} \tag{11}$$

$$\tilde{\rho}_{k|k-1}(\tilde{n}) = \sum_{j=0}^{\tilde{n}} \tilde{\rho}_{\Gamma,k}(\tilde{n} - j) \sum_{l=j}^{\infty} C_l^j \tilde{\rho}_{k-1}(l)(1-\phi)^{l-j}\phi^j \tag{12}$$

where

$$C_l^j = \frac{j!}{l!(j-l)!}$$

$$\phi = \frac{p_{S,k}^{(1)} \sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} + p_{S,k}^{(0)} N_{k-1}^{(0)}}{\sum_{i=1}^{J_{k-1}} w_{k-1}^{(i)} + w_k^b + N_{k-1}^{(0)}}.$$

B. Update

Given the predictions in (5), (10), (11), and (12), we may calculate the posterior PHD, number of clutter generators, and cardinality distribution by

$$\begin{aligned} v_k^{(1)}(x) = & \sum_{i=1}^{J_{k|k-1}} w_{k,m}^{(i)} \mathcal{N}\left(x; m_{k|k-1}^{(i)}, P_{k|k-1}^{(i)}\right) \\ & + \sum_{z \in Z} \sum_{i=1}^{J_{k|k-1}} w_{k,s}^{(i)}(z) \mathcal{N}\left(x; m_k^{(i)}(z), P_k^{(i)}\right) \\ & + \sum_{z \in Z} w_{k,b}(z) \mathcal{N}\left(x; \tilde{m}_k(z), \tilde{P}_k(z)\right) \end{aligned} \tag{13}$$

$$N_k^{(0)} = N_{k|k-1}^{(0)}(1 - p_{D,k}^{(0)})\tilde{\chi} + \sum_{z \in Z} \frac{\lambda_{k|k-1}^{(0)} \alpha_k(z)}{D_1(z)} \tag{14}$$

$$\tilde{\rho}_k(\tilde{n}) = \begin{cases} 0, & \tilde{n} < |Z_k| \\ \frac{\tilde{\rho}_{k|k-1}(\tilde{n}) \Upsilon_k^{(0)}[\tilde{v}_{k|k-1}, Z_k](\tilde{n})}{\langle \tilde{\rho}_{k|k-1}, \Upsilon_k^{(0)} \rangle}, & \tilde{n} \geqslant |Z_k| \end{cases} \tag{15}$$

In (13) above, we have calculated the measurement updated means and covariances using the extended Kalman Filter (EKF), according to

$$m_k^{(i)}(z) = m_{k|k-1}^{(i)} + K_k^{(i)}(z - \hat{z}_{k|k-1}^{(i)}) \quad (16)$$

$$P_k^{(i)} = P_{k|k-1}^{(i)} - K_k^{(i)} H_k^{(i)} P_{k|k-1}^{(i)} \quad (17)$$

$$q_k^{(i)}(z) = \mathcal{N}(z; \hat{z}_{k|k-1}^{(i)}, S_{k|k-1}^{(i)}) \quad (18)$$

$$\hat{z}_{k|k-1}^{(i)} = h(m_{k|k-1}^{(i)}, \boldsymbol{x}_{k,s}) \quad (19)$$

$$K_k^{(i)} = P_{k|k-1}^{(i)} \left(H_k^{(i)}\right)^T \left(S_{k|k-1}^{(i)}\right)^{-1} \quad (20)$$

$$S_{k|k-1}^{(i)} = H_k^{(i)} P_{k|k-1}^{(i)} \left(H_k^{(i)}\right)^T + \sigma_\theta^2 \quad (21)$$

$$H_k^{(i)} = \left.\frac{\delta h(x, \boldsymbol{x}_{k,s})}{\delta x}\right|_{x=m_{k|k-1}^{(i)}} \quad (22)$$

Note that this could easily be replaced by another nonlinear filter such as an unscented or cubature Kalman filter if desired. The weights of the Gaussian components in the updated mixture are calculated by

$$w_{k,m}^{(i)} = (1 - p_{D,k}^{(1)})\tilde{\chi} w_{k|k-1}^{(i)} \quad (23)$$

$$w_{k,s}^{(i)}(z) = \frac{p_{D,k}^{(1)} w_{k|k-1}^{(i)} q_k^{(i)}(z)}{D_1(z)} \quad (24)$$

$$w_{k,b}(z) = \frac{w_k^b/2\pi}{D_1(z)} \quad (25)$$

where,

$$D_1(z) = \lambda_{k|k-1}^{(0)} \alpha_k(z) + p_{D,k}^{(1)} \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} q_k^{(i)}(z) + \frac{w_k^b}{2\pi} \quad (26)$$

$$\tilde{\chi} = \frac{1}{D_2} \frac{\langle \tilde{\Upsilon}_k^1[\tilde{v}_{k|k-1}, Z_k], \tilde{\rho}_{k|k-1} \rangle}{\langle \tilde{\Upsilon}_k^0[\tilde{v}_{k|k-1}, Z_k], \tilde{\rho}_{k|k-1} \rangle} \quad (27)$$

$$\tilde{\Upsilon}_k^u[\Phi_{k|k-1} Z_k](\tilde{n}) = \begin{cases} 0, & \tilde{n} < |Z_k| + u \\ P_{|Z_k|+u}^{\tilde{n}} \Phi_{k|k-1}^{\tilde{n}-(|Z_k|+u)}, & \tilde{n} \geqslant |Z_k| + u \end{cases} \quad (28)$$

$$\Phi_{k|k-1} = 1 - \frac{1}{D_2}\left(p_{D,k}^{(1)} \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} + \lambda_{k|k-1}^{(0)} \right) \quad (29)$$

$$D_2 = \sum_{i=1}^{J_{k|k-1}} w_{k|k-1}^{(i)} + w_k^b + N_{k|k-1}^{(0)} \quad (30)$$

$$P_{|Z_k|+u}^{\tilde{n}} = \frac{\tilde{n}!}{(\tilde{n} - (|Z_k| + u))!} \quad (31)$$

Finally, the means $\tilde{m}_k(z)$ and covariances $\tilde{P}_k(z)$ of the updated birth terms in (13) are approximated according to the method in Section 3.7.2 of [11]. For the simulations in section V, we assumed a prior target course of $z - \pi$ so that targets are initially assumed to be moving directly towards the sensor along the line of bearing.

IV. Bootstrap GMCPHD Filter

The λ-GMCPHD filter does not propagate the cardinality distribution of real targets, but the joint cardinality distribution of real and clutter targets. This leads to increased variability in the estimated number of real targets compared to the conventional GMCPHD filter, which propagates the cardinality distribution of real targets only. By running a λ-GMCPHD and conventional GMCPHD filter in parallel, we can utilise the strengths of both filters to improve the overall performance. A simple way to do this is to use λ-GMCPHD to estimate the clutter density only, and

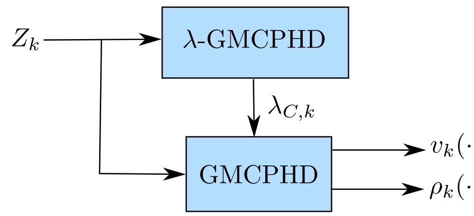

Fig. 1: Bootstrap GMCPHD filter

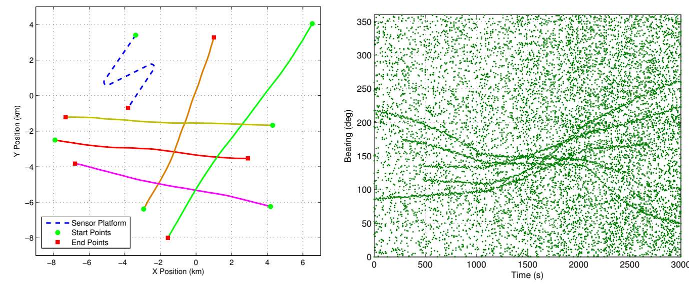

Fig. 2: Scenario geometry and example bearing measurements with time varying clutter rate

the conventional GMCPHD to estimate the intensity and cardinality distribution of real targets. On each scan, the λ-GMCPHD filter is updated first, and its estimate of the current clutter density calculated as $\lambda_{C,k} = N_k^{(0)} p_{D,k}^{(0)}$ which is then used in the update of the conventional GMCPHD filter, as illustrated in Figure 1.

V. Simulation Results

In this section we examine the performance of the three versions of the GMCPHD filter applied to a bearings-only multi-target tracking problem. The bootstrap GMCPHD filter is compared to the λ-GMCPHD and the conventional GMCPHD filter with various fixed clutter rate parameters. The performance of each filter is analysed by carrying out 500 Monte Carlo runs and plotting the average Optimal Sub-Pattern Assignment (OSPA) distance [12] between the estimated and true states, with order 1 and cutoff 4 km.

The scenario consists of a single bearings-only sensor on board a manoeuvring platform, and up to five targets with nearly constant velocity ($\sigma_v = 0.005$ m/s). Two targets are present at the beginning, with another three appearing shortly thereafter, and two disappearing late in the scenario. The detection probability and measurement noise are both known constants, with values of 0.95 and 1° respectively. The number of clutter measurements is Poisson, and the spatial distribution is uniform across all possible bearings. Two different clutter backgrounds are tested, one with a constant rate of 30, and another with a variable rate starting at 20 and increasing to 40 during the scenario. The scenario geometry and an example of the measurements for the variable clutter rate case are shown in Figure 2.

For the case of constant clutter rate, we compare the performance of the bootstrap and λ-GMCPHD with a conventional GMCPHD. The latter is tested under three different clutter parameter settings; 30 (matched), 40 (too high), and 20 (too low). For the λ-GMCPHD filter, parameter values of $p_{D,k}^{(0)} = 0.5$, $p_{S,k}^{(0)} = 0.98$, and $N_{\Gamma,k}^{(0)} = 1$ were used. This is tuned to achieve best performance for a clutter rate of 25, which is slightly offset from the true rate of 30.

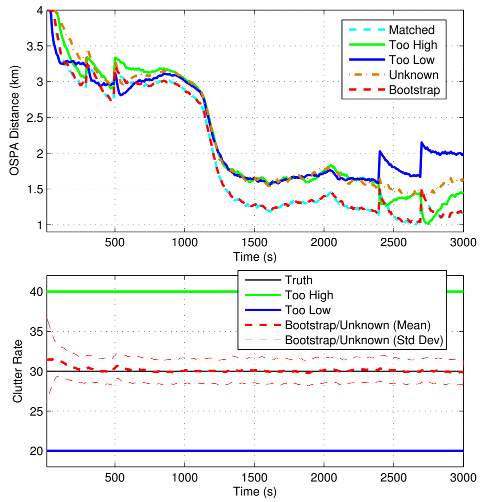

Fig. 3: Results for the fixed clutter rate scenario

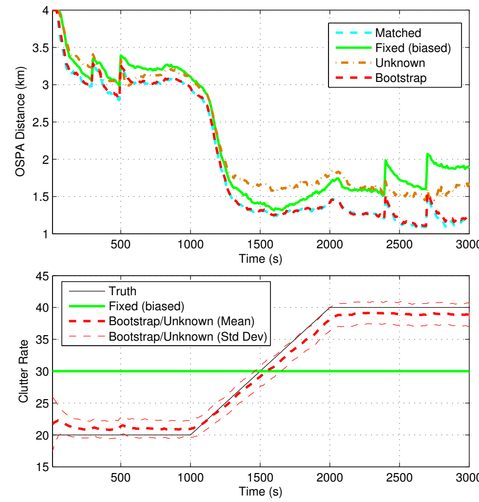

Fig. 4: Results for the variable clutter rate scenario

The OSPA and clutter rate estimates/parameters for the constant case are shown in Figure 3. As expected, the matched filter performs best, and the unmatched filters perform worse due to the bias introduced by the incorrect clutter rate setting. When the clutter rate is set too low, the filter responds more quickly to new target appearance, but more slowly to target disappearance, because the filter is more likely to explain measurements as originating from targets instead of clutter. The opposite effect occurs when the clutter rate parameter is set too high. Interestingly, the λ-GMCPHD alone (labelled 'unknown' on the plots), performs no better on average than the unmatched filters, due to the lack of cardinality information for real targets. The bootstrap GMCPHD filter resolves this problem, with performance very close to the matched filter.

For the variable clutter rate scenario, we compare the two adaptive methods with two parameterisations of the conventional GMCPHD filter, one of which assumes a fixed clutter rate of 30, and the other is the ideal but unrealistic case in which the clutter rate is matched to the exact true value on each scan. For the λ-GMCPHD filter, the clutter parameters are the same as those used in the constant clutter rate case.

Figure 4 shows the OSPA for the four filters and the clutter rate estimation performance. Again, due to the lack of target cardinality information, the λ-GMCPHD alone performs no better on average than the unmatched conventional GMCPHD. The bootstrap GMCPHD gives a significant improvement over the unmatched filter, with performance very similar to the ideal matched filter. The reason for this is that the λ-GMCPHD filter is capable of automatically responding to changes in clutter rate over time, as illustrated in Figure 4.

Some bias in the estimated clutter rate is evident from Figure 4, which is due to the fact that the parameters of the clutter model are tuned to give best performance for true clutter rates of around 25. Hence, when the true rate is lower, the estimate will be biased high, and when the true rate is higher, the estimate will be biased low. However, these small biases were not detrimental to the performance of the filter, and the average OSPA is almost identical to the unrealistic case in which the clutter rate is matched exactly to the truth. Clearly, a fixed clutter rate filter could not provide this level of performance, unless its parameters were being manually updated as the clutter rate changes.

VI. Conclusion

In this paper we have compared the performance of the conventional GMCPHD filter, the λ-GMCPHD filter for tracking with unknown clutter rates, and a bootstrap filter combining these two methods. Monte Carlo simulations on a bearings-only multi-target filtering problem with both fixed and time-varying clutter rates were carried out. As expected, simulation results show that the conventional GMCPHD gives the best performance if provided with an accurate clutter rate parameter setting. When the clutter rate is a-priori unknown, the bootstrap GMCPHD filter significantly outperforms both the λ-GMCPHD, and the conventional GMCPHD with unmatched clutter rate, with performance very close to that of the matched conventional GMCPHD filter.

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