Novel Data Association Schemes for the Probability Hypothesis Density Filter

Kusha Panta, Ba-Ngu Vo and Sumeetpal Singh

***Abstract*— The probability hypothesis density (PHD) filter is a practical alternative to the optimal Bayesian multi-target filter based on finite set statistics. It propagates the PHD function, a first order moment of the full multi-target posterior density. The peaks of the PHD function give estimates of the target states. However, the PHD filter keeps no record of target identities and hence does not produce track valued estimates of individual targets. In this paper, we propose two different schemes according to which the PHD filter can provide track valued estimates of individual targets. Both schemes use the probabilistic data-association functionality albeit in different ways. In the first scheme, the outputs of the PHD filter are partitioned into tracks by performing track-to-estimate association. The second scheme uses the PHD filter as a clutter filter to eliminate some of the clutter from the measurement set before it is subjected to existing data association techniques. In both schemes, the PHD filter effectively reduces the size of the data that would be subject to data association. In this paper, we consider the use of multiple hypothesis tracking (MHT) for the purpose of data association. This paper also discusses the performance of the proposed schemes and compare it with that of MHT.**

***Index Terms*— Multi-target Tracking, Random Sets, Probability Hypothesis Density (PHD) Filter, Multiple Hypothesis Tracking.**

I. Introduction

In multi-target tracking (MTT) problems, the number of individual targets and the measurements generated by targets change over time as both targets and clutter appear and disappear in the scene. One approach to tracking multiple targets, assuming each target moves independently of others, is to consider each target separately and track it with a separate filter. However this requires correct association of individual targets with their measurements [1], [2], [3]. Multiple hypothesis tracking (MHT) is a widely used technique that performs data association on a sequence of measurements and performs filtering on each data partition. MHT hypothesizes all possible data associations over time and uses measurements that arrive later in time to resolve the uncertainties in current associations. However, the complexity of the algorithm and inherent computational costs of such exhaustive data association are considerable. In practice, MHT uses various ad-hoc techniques to stop the number of hypotheses from growing exponentially with time. A computationally cheaper alternative to MHT is the joint probabilistic data association (JPDA) approach [4]. Instead of allowing all feasible associations to propagate ahead in time, JPDA considers associations that survive gating and combines these associations in proportion to their likelihoods. However, JPDA can only handle targets whose number is fixed over time and its performance is inferior to that of MHT [1], [2], [21]. In addition, MHT and JPDA both use extended Kalman filters (EKFs) for filtering the associated measurements and consequently suffer from drawbacks of the EKF. The EKF performs poorly when the non-linearity of target dynamics and/or the measurement process is severe. It also requires that the noise be additive Gaussian.

Another approach is to apply random finite sets (RFSs) to represent the collection of individual targets and measurements and recast the multi-target tracking problem in the Bayesian framework using finite set statistics (FISST) [6], [7], [8]. A RFS is, simply, a set-valued random variable just as a random vector is a vector-valued random variable. While the Bayesian recursion for the full multi-target posterior density of the set-valued random variable is computationally intractable, it can be approximated with a computationally cheaper alternative, called the probability hypothesis density (PHD) filter [7], [9]. The PHD filter recursively propagates the first order moment of the multi-target posterior density from which the estimates of the number of targets as well as the individual target states can be obtained. The first order moment of the multi-target posterior is known as the probability hypothesis density (PHD) function in the tracking literature. Sequential Monte Carlo (SMC) implementations of the PHD filter, called particle-PHD (or SMC-PHD) filter, have been proposed in [11], [12], [14], [15] to overcome the inherent intractability of the PHD filter since PHD recursion still consists of multiple integrals that have no closed form expressions in general. Moreover, it has been shown that the PHD filter provides good estimates of multi-target states. However, the existing implementations of the PHD filter do not incorporate individual target identities and hence do not provide the track-valued estimates.

This paper addresses the issue of obtaining the estimates of individual target tracks using the PHD filter. First, we present a novel way of associating the target estimates that are obtained with the PHD filter. The proposed 'PHD-with-association filter' treats the target state estimates given by the SMC-PHD filter as new measurement sets and performs partition on a batch of the estimates over time by performing track-to-estimate association in order to produce individual target

Manuscript received March 21, 2005; revised February 28, 2006 and April 13, 2006.

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tracks. The size of the target estimate set generated by the PHD filter is much smaller than that of the actual measurement set as the PHD filter outputs are almost free of clutter. This results in a substantial reduction on the computation and memory required to produce target tracks that would have been required if data-association were to be performed directly on the measurement sets. Furthermore, individual tracks produced by the track-to-estimate association already consist of target estimates and hence require no further filtering on them. For illustration purposes, we present simulation results for the 'PHD-with-association filter' and benchmark its performance against that of a MHT tracker. We show in Section IV that this scheme performs comparatively well against the benchmark algorithm, MHT. The SMC implementation of the PHD filter means this technique will be applicable to multi-target tracking problems with non-linear and/or non-Gaussian target and measurement models.

This paper also proposes another scheme named MHT-with-PHD clutter filter that considers the use of the PHD filter as a clutter filter to eliminate the measurements that are unlikely to have originated from targets before measurements are tracked with the existing data-association algorithms like MHT. This can be seen as a gating operation on a global level and will eliminate most of the clutter generated measurements. Section IV will present simulation results of the MHT-with-PHD clutter filter for a given tracking tracking scenario.

The issue of data association with the PHD filter has also been considered in [17] independent of the work presented in [16] where the core ideas of this paper first appeared. In [17], the peak-to-track association is considered as a two-dimensional assignment problem and does not use the estimates that will arrive subsequently to reduce the uncertainty in the association. Moreover, the peak-to-track association is primarily intended for linear/Gaussian tracking scenarios. Further discussion on the contributions of [17] will be presented later in Subsection IV-C.

The rest of the paper is organized as follows. Subsection II-A describes the multi-target model considered in this paper. Subsection II-B defines the PHD function and the PHD filter. Subsection II-C presents an overview of multi-target tracking with MHT. Section III proposes a novel scheme for adding target tracking capability to the PHD filter. Furthermore, we present an alternative scheme in which the PHD filter can be used to improve the performance of the MHT. Section IV presents the simulation results for the proposed schemes. Section IV also discusses the performance of both schemes proposed in Section III and in particular that of the 'PHD-with-association filter'. Finally, section V presents concluding remarks of this paper.

II. Multi-Target Tracking/Filtering Background

This section describes a general multi-target tracking/filtering problem and the description of two different approaches to multi-target tracking/filtering using the PHD and the MHT filters.

A. Multi-Target Model

In a multi-target tracking problem, targets appear and disappear randomly. New targets appear in the scene either due to spontaneous target birth or because of targets spawning. The number of targets born at each instant follows Poisson distribution with a mean of λ_b. A target present at time k may not survive to the next time step and die. The target death is modelled with a probability of $1 - e_{k|k-1}(x_{k-1})$, where $e_{k|k-1}(x_{k-1})$ represents the probability that a target present at time step $(k-1)$ will survive to the time step k. For the duration the target is present, it moves according to a Markov dynamic model

$$x_{k,i} \sim f_{k|k-1}(\cdot|x_{k-1,i}), \tag{1}$$

and generates observations according to

$$y_{k,i} \sim g_k(\cdot|x_{k,i}) \tag{2}$$

with a probability $p_D(x_k)$, where $p_D(x_k)$ gives the probability of a target being detected. We assume that a target generates only one observation at maximum.

At time k, let N_k be the number of targets present with states $x_{k,1},\ldots,x_{k,N_k}$, and M_k the corresponding number of measurements received. Let

$$X_k = \{x_{k,1},\ldots,x_{k,N_k}\} \subset E_s, \tag{3}$$

$$Y_k = \{y_{k,1},\ldots,y_{k,M_k}\} \subset E_o, \tag{4}$$

denote the set of targets and measurements received at time k. E_s and E_o represent the state and the observation spaces where individual targets and observations respectively lie. Some of M_k observations may be due to clutter. If $y_{k,i}$ is due to clutter, then $y_{k,i}$ follows the clutter probability density. The number of clutter points are assumed to be Poisson distributed with a mean of λ_c.

B. Multi-Target Filtering with the PHD Filter

Modelling the collection of individual targets and measurements with RFSs captures the time-varying nature of the number of targets present in the horizon and the number of measurements available at each time step. A RFS Ξ is a finite set-valued random variable, which can be generally described by a discrete probability distribution and a family of joint probability densities [5], [6]. Moreover, the discrete distribution characterizes the cardinality of Ξ (i.e., $|\Xi|$) whereas an appropriate density describes the joint distribution of the elements of Ξ for a given cardinality. In MTT, $|\Xi|$ gives the number of targets in the scene and each element $x \in \Xi$ corresponds to an individual target.

Finite set statistics (FISST) [6], [7], [8], [9] provides a rigorous framework for constructing the multi-target density of these RFSs and thus enables the formulation of the multi-target tracking problem in the Bayesian framework. Optimal Bayes multi-target filtering involves propagating the multi-target posterior density in time. However the inherent computational intractability in general means that we have to approximate the multi-target posterior density with its statistical moments and propagate the moments instead. Assuming the point process represented by the RFS is Poisson, its statistics are completely characterized by its first moment or the PHD function [6], [9]. The rest of this section introduces the PHD function and multi-target tracking/filtering with the PHD filter.

1) Probability Hypothesis Density: The probability hypothesis density (PHD) function D_Ξ is the 1st order moment of the RFS Ξ [7], [9] and is given by

$$D_\Xi(x) \equiv \mathbf{E}[\delta_\Xi(x)] = \int \delta_X(x) P_\Xi(dX) \tag{5}$$

where $\delta_\Xi(x) = \sum_{x\in\Xi}\delta_x$ and $\delta_\Xi(x)$ is the random density representation of Ξ. P_Ξ is the probability distribution of the RFS Ξ. Readers should see [7], [8], [9], [10], [12] for the detailed definitions of RFS and PHD function. The PHD D_Ξ of Ξ is an almost everywhere (a.e.) unique function on the space E_s where the individual targets exist. Its integration over a measurable subset $S \subseteq E_s$, i.e., $\int_S D_\Xi(x)\lambda(dx)$, yields the expected number of elements of Ξ that are present in S. Moreover, the N highest peaks of the PHD of Ξ, where N is the integer approximation to the estimated number of elements, give the estimates of the elements of Ξ. Provided the PHD function representing the targets at each time step is available, individual targets can be extracted from it.

2) The PHD filter: The PHD filter is a recursion of the PHD $D_{k|k}$ of the multi-target posterior $p_{k|k}$ and can be expressed in terms of the *prediction* and *update* operators. Assuming the RFS is Poisson, the recursion propagating $D_{k|k}$ in terms of the prediction operator $\Phi_{k|k-1}$ and the update operator Ψ_k follows [9]

$$D_{k|k} = (\Psi_k \circ \Phi_{k|k-1})(D_{k-1|k-1})$$

These operators are defined as follows:

$$(\Phi_{k|k-1}\alpha)(x) = \int \phi_{k|k-1}(x,\xi)\alpha(\xi)\lambda(d\xi) + \gamma_k, \tag{6}$$

$$(\Psi_x\alpha) = \left[\upsilon(x) + \sum_{y\in Y_k}\frac{\psi_{k,y}(x)}{\kappa_k(y) + \langle\psi_{k,y},\alpha\rangle}\right]\alpha(x), \tag{7}$$

for any (integrable) function α on E_s, where

$$\begin{aligned}
\phi_{k|k-1}(x,\xi) &= e_{k|k-1}(\xi)f_{k|k-1}(x|\xi) + b_{k|k-1}(x|\xi),\\
\upsilon(x) &= 1 - p_D(x),\\
\psi_{k,y}(x) &= p_D(x)g_k(y|x),\\
\kappa_k(y) &= \lambda_k c_k(y),\\
\langle f,h\rangle &= \int f(x)h(x)\lambda(dx)
\end{aligned}$$

and γ_k denotes the PHD of the RFS Γ_k of targets which appear spontaneously; $b_{k|k-1}(\cdot|\xi)$ denotes the PHD of the RFS $B_{k|k-1}(\{\xi\})$ spawned by a target with previous state ξ; $e_{k|k-1}(\xi)$ denotes the probability that the target still exist at time k given that it had previous state ξ; $f_{k|k-1}(\cdot|\cdot)$ denotes the transition probability density of individual targets; $g_k(\cdot|\cdot)$ denotes the likelihood of individual targets; $c_k(\cdot)$ denotes the clutter probability density; λ_k denotes the average number of Poisson clutter points per time step; and $p_D(\cdot)$ denotes the probability of detection. The details on the derivation of these density functions and likelihood functions from the underlying models of the sensors, individual targets dynamics, target births and deaths are found in [6], [7]. It should be noted that the PHD filter will reduce to a standard Bayesian filter that propagates the posterior density of a target when there is one target in the scene with no target birth or death.

The PHD filter is a computationally cheaper alternative to the optimal multi-target filter. However it still involves computation of multiple integrals that have no closed form solutions in general, making it computationally intractable. The sequential Monte Carlo (SMC) implementation of the PHD filter, known as the particle-PHD (or SMC-PHD) filter has recently been proposed in [11], [12]. Similar SMC implementations of the particle-PHD filter have been independently proposed in [14], [15]. Convergence properties of the particle-PHD filter have been established in [12]. A brief description of the generic particle-PHD filter is given below.

Particle-PHD (or SMC-PHD) filter

Given a set of L_k particles $\{w_k^{(i)}, x_k^{(i)}\}_{i=1}^{L_k}$ approximating the PHD $D_{k|k}$ at $k=0$, the PHD at time step $k>0$ is obtained by:

- *prediction step*: draw samples according to the proposal densities $q_k\left(\cdot|x_{k-1}^{(i)}, Y_k\right)$ and $p_k(\cdot|Y_k)$

$$\widetilde{x}_k^{(i)} \sim \begin{cases} q_k\left(\cdot|x_{k-1}^{(i)}, Y_k\right), & i = 1,...,L_{k-1}\\ p_k(\cdot|Y_k), & i = L_{k-1}+1,...,L_{k-1}+J_k\end{cases}$$

  and compute associated weights as

$$\widetilde{w}_{k|k-1}^{(i)} = \begin{cases} \dfrac{\phi_{k|k-1}(\tilde{x}_k^{(i)}, x_{k-1}^{(i)})}{q_k\left(\tilde{x}_k^{(i)}|x_{k-1}^{(i)}, Z_k\right)} w_{k-1}^{(i)}, & i = 1 : L_{k-1}\\ \dfrac{1}{J_k}\dfrac{\gamma_k\left(\tilde{x}_k^{(i)}\right)}{p_k\left(\tilde{x}_k^{(i)}|Z_k\right)}, & i = L_{k-1}+1 : L_{k-1}+J_k\end{cases}$$

- *update step*: update each of the weights

$$\widetilde{w}_k^{(i)} = \left[\upsilon(\widetilde{x}_k^{(i)}) + \sum_{y\in Y_k}\frac{\psi_{k,y}(\widetilde{x}_k^{(i)})}{\kappa_k(y) + C_k(y)}\right]\widetilde{w}_{k|k-1}^{(i)}.$$

  where

$$C_k(y) = \sum_{j=1}^{L_{k-1}+J_k}\psi_{k,y}(\widetilde{x}_k^{(j)})\widetilde{w}_{k|k-1}^{(j)}.$$

- *resampling step*: obtain $\left\{w_k^{(i)}\widetilde{N}_{k|k}, x_k^{(i)}\right\}_{i=1}^{L_k}$ by resampling $\left\{\widetilde{w}_k^{(i)}\widetilde{N}_{k|k}, \widetilde{x}_k^{(i)}\right\}_{i=1}^{L_{k-1}+J_k}$ where $\widetilde{N}_{k|k} = \sum_{j=1}^{L_{k-1}+J_k}\widetilde{w}_k^{(j)}$.

The recursion described above only gives a particle approximation of the PHD function at each time step, from which the estimate of number of targets and their individual state estimates have to be obtained. The estimate of the number of targets is given by $\widetilde{N}_k$, where $\widetilde{N}_k$ is integer approximation to

$\widetilde{N}_{k|k}$. At each time step, a clustering program such as the K-means clustering algorithm [19] can be used to determine the centers of clusters that form the particle approximation of the PHD function. These centers represent the peaks of the PHD function, which can be used as estimates of individual target states. As a result, the peak extraction technique employed to obtain the target states estimates from the given PHD approximation also has bearing on the performance of the PHD filter in MTT.

C. Multi-target Tracking with Multiple Hypothesis Tracking

Given the set of observations, multi-target tracking based on data association requires correct partitioning of measurements amongst individual targets and clutter. A simple approach would be to find the most probable association of measurements with individual targets at each time step. Instead of making the decision on the most probable data partitioning at each time step, the MHT hypothesizes several possible partitioning of measurements and propagates these hypotheses along time so that uncertainty in the correct partitioning can be resolved on arrival of subsequent data [1], [2], [3], [20], [21], [22], [23]. However the number of hypotheses can grow exponentially over time and the required computational costs could render the implementation of MHT infeasible. *Gating* and *pruning* techniques are *ad-hoc* methods commonly used to limit the number of track hypotheses at each time step by eliminating the unlikely data partitions.

Detailed descriptions of multi-target tracking based on MHT can be found in [1], [21], [22]. This section provides only an overview of a track-oriented implementation of the MHT algorithm that is used for illustration purposes later in section IV. A track-oriented MHT is chosen over a hypothesis-oriented one as the track-oriented MHT is simpler to implement and results in a smaller number of hypotheses [22], [23]. In the track-oriented approach, each track represents a collection of assigned measurements over time that are likely to have originated from the same target. Collections of tracks that originate from the same root and represent the same target are referred as track hypotheses while each track is a trace of successive branches from the root of the tree to a leaf. Once the existing track hypotheses are updated with new measurements, track associations that have low probabilities are pruned before the new hypotheses are formed and propagated to the next time step. The probability of each track hypothesis is determined by its likelihood score, often maintained as the log-likelihood ratio (LLR).

1) Overview of track-oriented MHT: To begin with, tracks are initiated for all measurements. Clusters of tracks are formed so that tracks in one cluster share observation amongst each other and do not share observations with tracks from other clusters. Clusters of tracks are formed in order to reduce the complexity of the algorithm during data output. Figure 1 shows a block diagram of a practical implementation of the track-oriented MHT. Given a number of track hypotheses at time step $(k-1)$, MHT allows measurement prediction for tracks in each track hypothesis. Track-to-measurement association is performed on each track hypothesis with the noisy

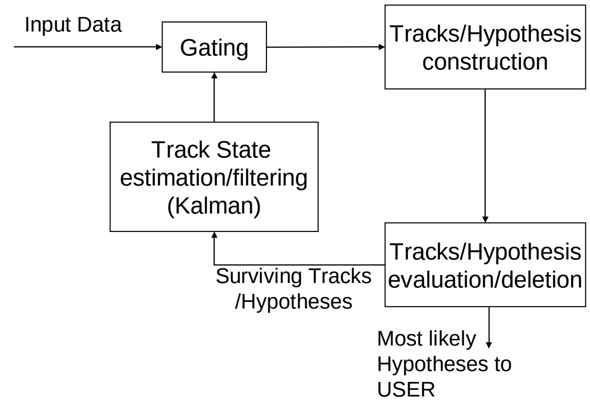


Fig. 1. An overview of track-oriented MHT algorithm.

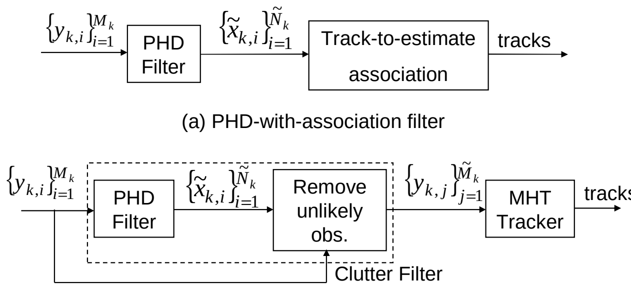
measurements available at time step k. For m_k measurements that fall within the gate of the prediction of a track, m_k association hypotheses are formed where each association hypothesis corresponds to the association of an existing track with one of m_k measurements. When no measurement falls with the prediction of an existing track, a target detection-miss is noted and the track is propagated ahead. A new track is started for each measurements that are not gated with any of the existing tracks. Upon the formation of each association track hypothesis, its score or LLR is initialized or updated.

Pruning of tracks is performed at the track level based on their LLRs and the number of consecutive target detection-misses. Association hypotheses with N (usually three or more) consecutive miss-detections or with LLRs smaller than a chosen *threshold* are deleted. Association hypotheses that have at least N (usually three or more) target detections are considered to be *confirmed* tracks. Confirmed track hypotheses are also subject to N-scan pruning on a global level. The choice of the actual parameter values used in pruning depend on the tracking scenario. The association hypotheses that survive pruning are updated with their respective gated observations and propagated to the next time step as tracks.

For each target, there may exist multiple tracks representing multiple assignments of measurements over the subsequent time steps. All tracks that are started by the same target form a tree structure with a common root. The MHT chooses the most likely track from every tree to produce a collection of track hypotheses called a global hypothesis. Several global hypotheses may be formed in order to find the combination of tracks that are most likely to represent individual targets. Hence the number of global hypotheses that may be formed will depend on the total number of tracks. Clustering of tracks will help to reduce the number of global hypotheses as global hypotheses from one cluster may be chosen independent of others. Only confirmed tracks are considered for the selection of global hypotheses. At each time step, the sequence of measurements represented by a track in the global hypotheses is filtered to produce the track estimates of targets present in the horizon. MHT uses EKF for prediction and filtering on the tracks.

III. Multi-Target Tracking with the PHD Filter

The PHD filter introduced in Section II-B above only gives a set of state estimates of individual targets that are present in the scene at any time step k, i.e. $\widetilde{X}_k$. It keeps no records of the target identities and hence does not produce tracks followed by individual targets over time. However some data-association functionalities of MHT can be incorporated with the PHD filter to produce the track-valued estimates of individual targets. This section will present two different methods for doing so. We use the word 'tracks' to denote the trajectories followed by individual targets and the 'point state estimates' to denote the estimates of the target states at individual time steps.



(b): MHT-with-PHD clutter filter

Fig. 2. Multitarget Tracking with the PHD filter.

A. Scheme One: PHD-with-association filter

This approach takes the outputs of the PHD filter and performs data association on them to produce track-valued estimates of targets (see Figure 2(a)). At time step k, the PHD filter provides an estimate of the number of targets present in the scene and their states. Assuming these estimates are sufficiently accurate, we may regard the output of the PHD filter as defining a new observation model given by

$$\widetilde{y}_{k,i} = \widetilde{x}_{k,i} = x_{k,i} + n_{k,i}, \quad i = 1, \ldots, \widetilde{N}_k \tag{8}$$

and

$$\widetilde{Y}_k = \{\widetilde{y}_{k,i}\}_{i=1}^{\widetilde{N}_k}$$

where $\widetilde{N}_k$ is the estimate of the number of targets present at time step k and $\widetilde{Y}_k$ is the new observation set. The error in the estimate $n_{k,i} = \widetilde{x}_{k,i} - x_{k,i}$ can be regarded as noise. Thus, regardless of the fact that the original observation process is non-linear and/or non-Gaussian, the data-association functionality is given a linear observation process given by Eq. (8). Implicit in this scheme is the assumption that $\widetilde{N}_k \approx N_k$. There are a number of ways to estimate the statistics of the noise $n_{k,i}$ [24], [25]. In this paper, we assume $n_{k,i}$ is a zero-mean Gaussian process with variance $\widetilde{Q}_{k,i}$ that can be estimated from the particle approximation of the PHD recursion. In simulations, we find that this scheme works well even though the true distribution of $n_{k,i}$ is not Gaussian.

This scheme uses the particle-PHD filter first to recursively produce $\widetilde{X}_k$ from the set of noisy observations Y_k available at time step k. Batches of $\widetilde{X}_k, k \geq 0$ are then subjected to data association to produce individual target tracks taking $\widetilde{X}_k$ as the new observation set. This scheme can be summarised as follows:

The PHD-with association filter

Step 1: Initialize PHD $D_{k|k}$ at $k=0$ with a set of L_k particles $\{w_k^{(i)}, x_k^{(i)}\}_{i=1}^{L_k}$,

Step 2: For $k \geq 1$, obtain $\{w_k^{(i)}, x_k^{(i)}\}_{i=1}^{L_k}$ approximating $D_{k|k}$ recursively as

- *prediction step*: draw samples according to the proposal densities $q_k\left(\cdot|x_{k-1}^{(i)}, Y_k\right)$ and $p_k\left(\cdot|Y_k\right)$

$$\widetilde{x}_k^{(i)} \sim \begin{cases} q_k\left(\cdot|x_{k-1}^{(i)}, Y_k\right), & i = 1, ..., L_{k-1} \\ p_k\left(\cdot|Y_k\right), & i = L_{k-1}+1, ..., L_{k-1}+J_k \end{cases}$$

  and compute associated weights as

$$\widetilde{w}_{k|k-1}^{(i)} = \begin{cases} \frac{\phi_{k|k-1}(\widetilde{x}_k^{(i)}, x_{k-1}^{(i)})}{q_k\left(\widetilde{x}_k^{(i)}|x_{k-1}^{(i)}, Y_k\right)} w_{k-1}^{(i)}, & i = 1, ..., L_{k-1} \\ \frac{1}{J_k} \frac{\gamma_k\left(\widetilde{x}_k^{(i)}\right)}{p_k\left(\widetilde{x}_k^{(i)}|Y_k\right)}, & i = L_{k-1}+1, ..., L_{k-1}+J_k \end{cases}$$

- *update step*: update each of the weights

$$\widetilde{w}_k^{(i)} = \left[\upsilon(\widetilde{x}_k^{(i)}) + \sum_{y \in Y_k} \frac{\psi_{k,y}(\widetilde{x}_k^{(i)})}{\kappa_k(y) + C_k(y)}\right] \widetilde{w}_{k|k-1}^{(i)}.$$

  where

$$C_k(y) = \sum_{j=1}^{L_{k-1}+J_k} \psi_{k,y}(\widetilde{x}_k^{(j)}) \widetilde{w}_{k|k-1}^{(j)}.$$

- *resampling step*: obtain $\left\{w_k^{(i)} \widetilde{N}_{k|k}, x_k^{(i)}\right\}_{i=1}^{L_k}$ by resampling $\left\{\widetilde{w}_k^{(i)} \widetilde{N}_{k|k}, \widetilde{x}_k^{(i)}\right\}_{i=1}^{L_{k-1}+J_k}$ where $\widetilde{N}_{k|k} = \sum_{j=1}^{L_{k-1}+J_k} \widetilde{w}_k^{(j)}$.

Step 3: Point-valued estimates of targets are obtained as follows:

- form $\widetilde{N}_k \left(= \text{round}\left(N_{k|k}\right)\right)$ clusters from $\left\{w_k^{(i)} \widetilde{N}_{k|k}, x_k^{(i)}\right\}_{i=1}^{L_k}$
- centers of the clusters gives target estimates $\{\widetilde{x}_{k,i}\}_{i=1}^{\widetilde{N}_k}$
- covariances of clusters give $\{\widetilde{Q}_{k,i}\}_{i=1}^{\widetilde{N}_k}$

Step 4: Given $\{\widetilde{x}_{i,k}\}_{i=1}^{\widetilde{N}_k}$, track-to-estimate association follows:

- *initialization*:
  1) create a new track hypothesis with initial state $\widetilde{x}_{k,i}$ and initial covariance
  2) create a new cluster of the target represented by each track
- *prediction*: for $k > 1$, propagate the state and the covariance of each track hypothesis according to equation (1)
- *gating*: for each measurement $\widetilde{y}_{k,i}$ (given by $\widetilde{x}_{k,i}$)
  1) create association hypothesis for every tracks it is associated with
  2) initialize a track if it is not associated with any tracks

3) note a target *miss-detection* on every tracks that have no measurement associated to it

- update existing clusters
- update of LLRs of each association hypothesis
- perform tracks confirmation and track-level pruning
- form global hypothesis formation from confirmed tracks
- perform N-scan pruning
- perform measurement update on tracks that survived pruning according to equation (8)
- tracks represented by the best global hypotheses is used for data output

The implementation of the track-to-estimate association is very similar to the track-oriented MHT introduced in section II-C. However other versions of MHT should be applicable provided the effect of the PHD filter on the measurements is reasonably accounted for.

The motivation behind the PHD-with-association filter scheme is that the size of the modified observation set $\widetilde{Y}_k$ given by Eq. (8) is smaller than that of the original observation set Y_k. This is because almost every observation in $\widetilde{Y}_k$ is associated with a target where as Y_k has additional clutter. As a result, a much smaller number of track hypotheses are created at each time step and the computational cost of the track-to-association will be smaller as well. Moreover, pruning of track hypotheses can be simpler than in the MHT due to the smaller number of hypotheses thus reducing the overall complexity and computational requirement of the MHT. Since a track hypothesis consists of estimates that are associated with a target over time, there is no need for further filtering on individual tracks. This scheme is readily available to tracking scenarios that have nonlinear/non-Gaussian state models.

B. Scheme Two: The MHT-with-PHD Clutter Filter

In the above method, the PHD filter was effectively used as a clutter pre-filter that feeds a modified observation set to the data-association functionality. We now describe how to use the PHD filter as a clutter filter but without modifying the observation process seen by the data association functionality. The aim would be to eliminate most of the measurements that are likely to be clutter so that the computational cost of the data association would be reduced. As a result, the number of false tracks picked by the MHT should decrease. MHT filter is considered for the purpose of data-association.

The proposed scheme is depicted by the block diagram in Figure 2(b). The PHD filter outputs $\widetilde{X}_k$ are used to define validation gates in the observation space. The observations that fall outside the gates are discarded as clutter and the reduced observation set is fed to the MHT filter to produce track-valued estimates of the targets. In summary, given an observation set $Y_k = \{y_{k,i}\}_{i=1}^{M_k}$ at time step k, the new observation set is given by

$$\widetilde{Y}_k = \{y_k^i : g_k(y_k^i|\widetilde{x}_{k,j}) > g_d \quad \text{for some } j\},$$

where $\widetilde{X}_k = \{\widetilde{x}_{k,j}\}_{j=1}^{\widetilde{N}_k}$ is the PHD filter output at time step k and g_d is the observation gate threshold. Provided the measurement noise is additive and Gaussian, the gating proposed here becomes very similar to the process of gating used in data association techniques. Approximating each $\widetilde{x}_{k,j} \in \widetilde{X}_k$, with its mean position $\widetilde{x}_{k,j}$ and covariance $\widetilde{Q}_{k,j}$, a measurement $y_{k,i}$ is included in $\widetilde{Y}_{k,i}$ if

$$(y_{k,i} - g_k(\widetilde{x}_{k,j}))\, S_k^{-1}\, (y_{k,i} - g_k(\widetilde{x}_{k,j}))' < G_{th}$$

where $S_k = H_{k,j}\widetilde{Q}_{k,j}H_{k,j}' + R_k$, R_k is the covariance of the measurement noise, G_{th} is the size of the observation gate and

$$H_{k,j} = \left.\frac{\partial g_k(x)}{\partial x}\right|_{x=\widehat{x}_{k,j}}.$$

In cases when noise is only additive, the unscented transform [26], [27] can be used instead.

The choice of the gate size g_d or G_{th} depends on the tracking scenarios as well as the accuracy of the target estimates given by the particle-PHD filter. As the approximation of the full posterior with its PHD requires high SNR [9], the size of g_d could be small. In effect, the PHD filter eliminates most of the observations that are unlikely to have originated from the targets. Hence, this clutter filter approach can be viewed as a way of performing gating on a global level.

The implementation of the MHT-with-PHD clutter filter can be summarized as follows:

MHT-with-PHD clutter filter

- *PHD filtering*: apply step 1-3 of PHD-with-association filter to obtain, $\widetilde{X}_k = \{\widetilde{x}_{k,j}\}_{j=1}^{\widetilde{N}_k}$
- *Global gating*: obtain $\widetilde{Y}_k$ from $Y_k = \{y_{k,i}\}_{i=1}^{M_k}$ as follows:

  for $i : 1, \cdots, M_k$
    for $j : 1, \cdots, \widetilde{N_k}$
      if $g_k(y_{k,i}|\widetilde{x}_{k,j}) > g_d$
        include $y_{k,i}$ in $\widetilde{Y}_k$
      end
    end
  end
- *Data association & tracking*: apply MHT on $\widetilde{Y}_k$

The main motivation behind the MHT-with-PHD clutter filter is to filter out most of the clutter generated measurements exploiting the fact that the target estimates given the PHD filter are fairly accurate. These estimates have been shown to outperform MHT in terms of their multi-target miss distances [16] (see figure 8 in subsection IV-A). This type of global gating should perform well in nonlinear/non-Gaussian case as this does not approximate the nonlinearity present in the target dynamics and/or measurement process. Further investigation is required to determine the merit of tradeoff between the added computational cost of the global gating and the reduction in that of the data association as a result of smaller observation set.

IV. Simulation Results

For illustration purposes, we consider a two-dimensional scenario, in which each target moves in the region $[-100, 100] \times [-100, 100]$ and can appear or disappear in the scene at any time. The number of targets at any time is unknown and time-varying. The target states consist of positions and velocities. Each target moves according to the following linear and Gaussian target dynamics,

$$x_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} x_{k-1} + \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix} \begin{bmatrix} v_{1,k} \\ v_{2,k} \end{bmatrix} \tag{9}$$

where $x_k = [p_k^x,\ \dot{p}_k^x,\ p_k^y,\ \dot{p}_k^y]'$, and $[p_k^x,\ p_k^y]'$ and $[\dot{p}_k^x,\ \dot{p}_k^y]'$ represent position and velocity of the target at time k. $[\cdot]'$ represents the transpose of a matrix $[\cdot]$ and $T = 1$ is the sampling period in unit time step. The process noise $v_{1,k}$ and $v_{2,k}$ are independent zero mean Gaussian noise with standard deviations of 1 and 0.1 respectively. An existing target has a survival probability of $e_{k|k-1} = 0.95$ and this probability is state independent. We assume that target birth follows a Poisson model with the intensity $0.2\mathcal{N}(\cdot|x_0, Q_b)$, where $\mathcal{N}(\cdot|x_0, Q_b)$ represent a normal distribution with a mean x_0, covariance Q_b and

$$x_0 = \begin{bmatrix} 0 \\ 3 \\ 0 \\ -3 \end{bmatrix}, \quad Q_b = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{10}$$

Figure 3 shows the ground truth positions of four tracks that

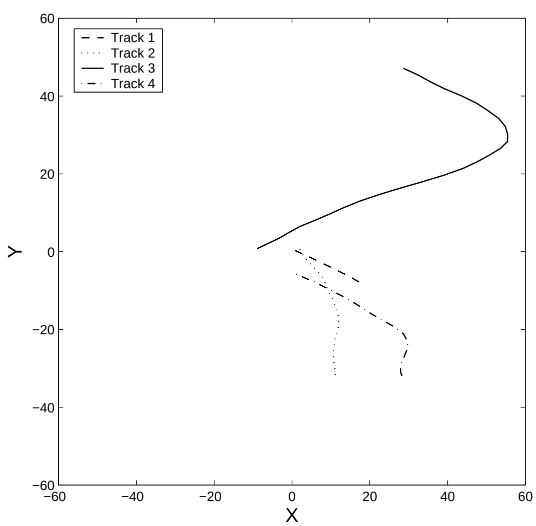


Fig. 3. plots of true positions of four tracks superimposed over 40 time steps.

appear around the origin and move radially outwards. The start and finish time of each track is displayed at figure 4 that plots the x and y components of the tracks separately against time.

The bearing θ_k and range r_k measurement of a target are generated by a sensor located at $[0, -100]^T$ and are given by

$$\theta_k = arctan\left(\frac{p_k^x}{p_k^y + 100}\right) + w_{1,k} \tag{11}$$

$$r_k = \sqrt{((p_k^x)^2 + (p_k^y + 100)^2)} + w_{2,k} \tag{12}$$

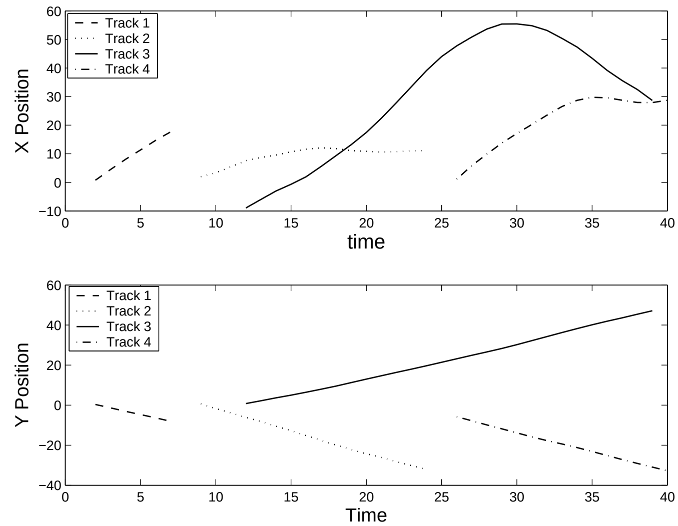


Fig. 4. plots of x and y components of true positions of 4 tracks given in figure 3 against time.

The measurement noise $w_{1,k}$ and $w_{2,k}$ are independent zero mean Gaussian noise with standard deviations of 0.05 and 2 respectively. Moreover, the measurement noise is also independent of the process noise. The probability of detection is assumed almost unity. Clutter is uniformly distributed over the observation space $[-100, 100] \times [-100, 100]$ with an average rate of r points per scan. At each time step, the target state estimates are extracted from the set of particles representing the updated PHD function by applying standard clustering techniques.

The particle implementation of the PHD filter uses 1000 particles per target. The proposal densities used for importance sampling are $q_k = \phi_{k|k-1}$ and $p_k = \mathcal{N}(\cdot|x_0, Q_b)$. This particular implementation of the track-oriented MHT uses an ellipsoid gate of size 9.21. Track hypotheses are confirmed once they have at least two target detections. A hypothesis track is deleted if it has either three or more consecutive target detection-misses. Confirmed tracks are also subject to N-scan pruning with N equals three. Tracks are not subjected to pruning on the basis of their likelihoods as this implementation of MHT can handle the number of track hypotheses without. For data output, the best global hypothesis is considered. G_{th} takes a value of 9.21 in the implementation of the MHT-with-PHD clutter filter.

A. *Simulation Results for Point State Estimates*

Figures 5 and 6 show plots of x and y position estimates of targets given by the PHD and the MHT filter. Similarly, Figure 7 shows the point state estimates of the targets given by the MHT-with-PHD clutter filter. The ratio of average clutter points to targets is 20. Figure 5 shows that the PHD filter provides good estimates of target positions and picks up a few false alarms. Figure 6 shows that the MHT filter picks a large number of false alarms in comparison to that produced by the PHD filter. However, Figure 7 shows that the number of false tracks can be reduced by using the PHD filter as a clutter filter. Quantitatively, the goodness of the point state estimates given is better measured in terms of the multi-target miss-distance between two finite sets representing the true target positions and their respective estimates.

Hoffman and Mahler [28] have shown the Wasserstein distance to be a good measure of the multi-target miss distance. Given the multi-target ground truth $X = \{x_1, \cdots, x_{|X|}\}$ and its point state estimate $\widetilde{X} = \{\widetilde{x}_1, \cdots, \widetilde{x}_{|\widetilde{X}|}\}$, the Wasserstein distance d_p^W is defined as

$$d_p^W(\widetilde{X}, X) = \min_C \sqrt[p]{\sum_{i=1}^{|\widetilde{X}|} \sum_{j=1}^{|X|} C_{i,j} \|\widetilde{x}_i - x_j\|^p}, \qquad (13)$$

where the minimum is taken over the set of all transportation matrices $C = \{C_{i,j}\}$; and each entry of the matrix C satisfies the followings: $C_{i,j} \geq 0$, $\sum_{i=1}^{|\widetilde{X}|} C_{i,j} = 1/|\widetilde{X}|$ and $\sum_{j=1}^{|X|} C_{i,j} = 1/|X|$. In this work, p takes a value of two.

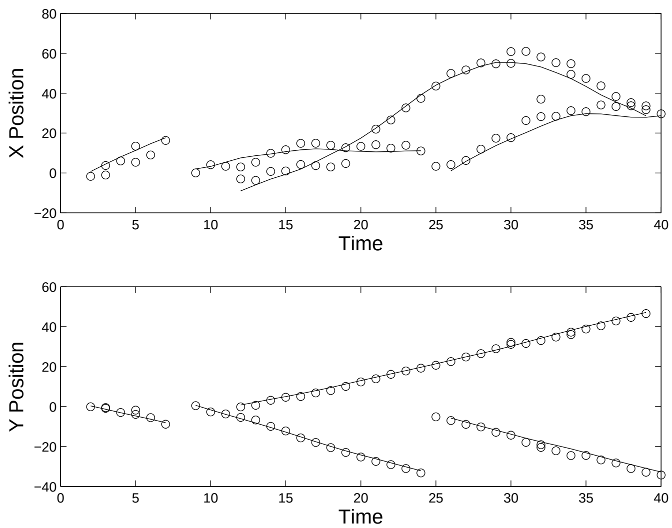


Fig. 5. plots of x and y components of the position estimates (circle) given by the PHD filter against time, superimposed with true target tracks (solid lines).

Figure 8 shows that the Wasserstein distance is small for the PHD estimates except at a few time steps when the number of target estimates does not match the number of true targets. However, the Wasserstein distance is considerably larger for the position estimates given by the MHT filter at most of the time steps. This is because the MHT filter picks a comparatively large number of clutter points and the Wasserstein distance penalizes the difference in cardinalities between two sets. The Wasserstein distance of the MHT filter can be reduced by using the PHD filter as a clutter filter. Figure 8 shows that the Wasserstein distance of the estimates given by the MHT-with-PHD clutter filter is smaller than that obtained with the MHT only. During the time steps (between time step 25 and 40) when the target estimates given the PHD filter are good, the MHT-with-PHD clutter filter works well. In this work, when both sets are empty, we assigned a zero distance to the multi-target miss distance. However, when one of the set is empty, the Wasserstein distance becomes infinite and might not be very meaningful in tracking. In practice, the

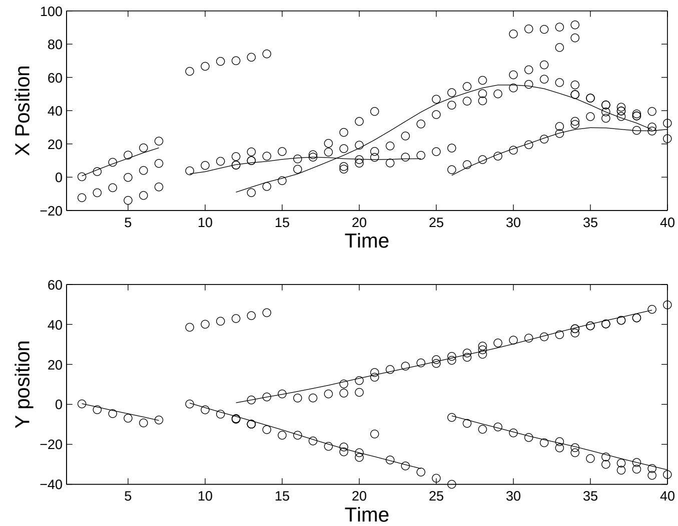


Fig. 6. plots of x and y components of the position estimates (circle) given by the MHT filter against time, superimposed with true target tracks (solid lines).

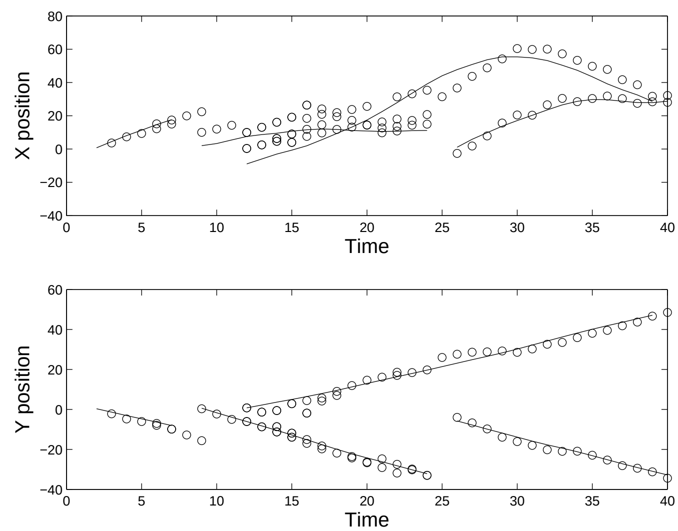


Fig. 7. Point State estimates obtained using MHT-with-PHD clutter filter superimposed with the true target tracks (solid line).

penalty one wants to impose on the tracker when one of the set is empty, depends on the user and the application.

B. Simulation Results for Track Valued Estimates

In this section, first we presents the track valued estimates of the targets given by the PHD-with-association filter, the scheme proposed in Subsection III-A. The noisy measurement set at each time step is filtered with the PHD filter and the outputs are then subjected to track-to-estimates association to form track valued estimates of individual targets. This is achieved by applying a track-oriented MHT on the outputs of the PHD filter. Each track in this case consists of a sequence state estimates of a target over time. The MHT algorithm treats the outputs of the PHD filter as the measurements generated according to the linear process given in equation (8).

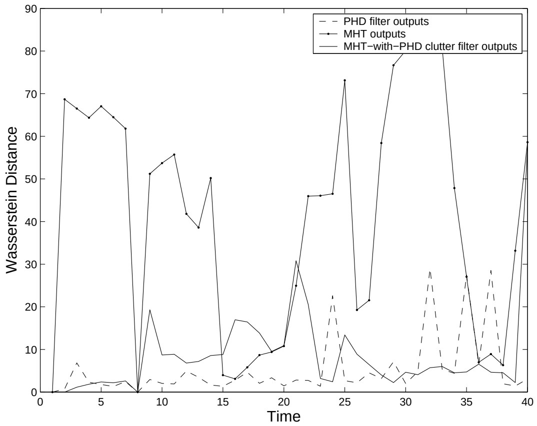


Fig. 8. The Wasserstein distance between the point estimate outputs of the PHD and MHT filter, and the ground truth

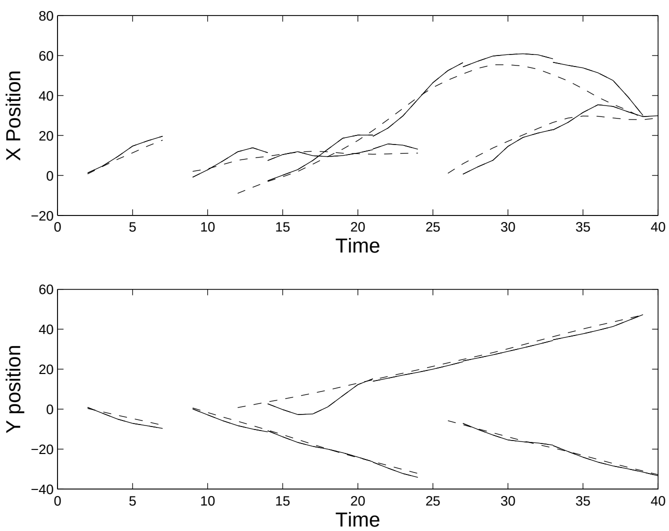


Fig. 9. Target tracks obtained using the track-oriented MHT filter on observation sets modified by the PHD filter (Scheme One) superimposed with the true target tracks (dashed line).

Figure 9 presents the tracks that are given by the PHD-with-association filter. The target tracks produced by the PHD-with-association filter matches closely with the true target tracks. There are no false tracks. It misses the start of the third and fourth track for two time steps. However, estimates of the target positions at these two time steps are available from the PHD filter and the end user should benefit from these position estimates as well as the individual target tracks obtained from these estimates.

Figure 10 shows the tracks produced by applying the MHT filter on the noisy observation set obtained according to the target and measurement dynamics presented earlier in Section IV. In addition to picking up the majority parts of true tracks, it picks a number of false tracks. False tracks are created when some of the randomly created clutter at successive time steps will gate with one another. One way of reducing the number of false alarms picked by the MHT filter is to increase the number of target detections needed in each hypothesis to be classified as a confirmed hypothesis. However

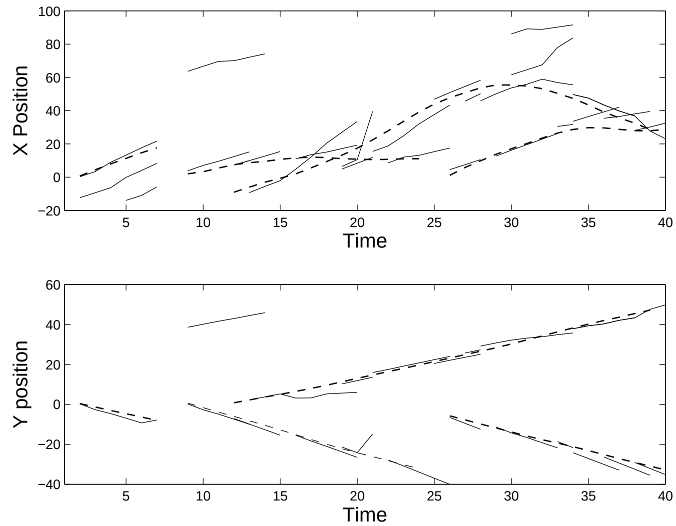


Fig. 10. Target tracks obtained using the track-oriented MHT filter on observation sets superimposed with the true target tracks (dashed line).

this will result in higher misses of true targets. The quality of tracks here also suffers from the drawbacks of the EKF that MHT uses for predictions and updates on track hypotheses during gating and measurement updates. It should also be noted that for MHT to pick up target tracks, the target must be present in the scene for at least a number of time steps equal to the number of target-detections (equals two in this simulation) needed for a track hypothesis to be confirmed.

Figure 11 shows the track estimates of the targets given by the MHT-with-PHD clutter filter. It shows that the number of false tracks picked by the MHT filter is smaller than the ones given by the stand alone MHT. However, it still picks up a larger number of false tracks than the PHD-with-association filter. Figure 11 shows that the MHT-with-PHD clutter filter gives good track estimates of targets provided point state estimates of individual targets given by the PHD filter are good.

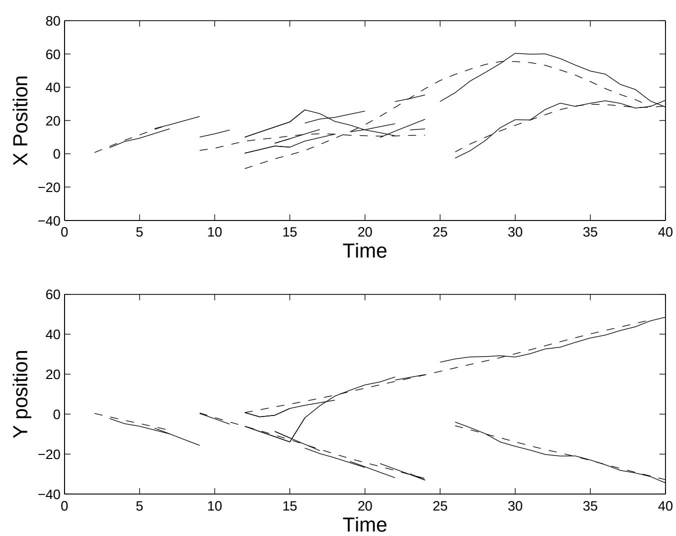


Fig. 11. Target tracks obtained using MHT-with-PHD clutter filter (Scheme Two) superimposed with the true target tracks (dashed line).

C. Discussion

Simulation results in Sections IV-A and IV-B show that the PHD filter can provide good track-valued estimates of the targets in addition to recursively providing good point state estimates of targets. The results also show that the PHD filter can also be used as a clutter filter to improve the performance of the MHT filter.

The PHD-with-association filter finds the most likely associations of target state estimates over time using primarily the individual target dynamics. In this scheme, some of the spurious clutter points that are picked up by the PHD filter can be eliminated during the track-to-estimate association by discarding any estimate that do not get associated with any of the estimates at the following time step. The main effect of the PHD filter in the PHD-with-association filtering is to provide fairly accurate estimates of the target states. For the purpose of track-to-estimate association, these estimates can be seen as the new measurements produced according to the linear measurement process given by equation (8). Thus it enables the track-to-estimates association functionality to do away with the need to use EKFs that, in general, do not work well in the presence of severe nonlinearity in the target dynamics and/or the measurement process.

Moreover, the PHD filter provides almost clutter free point state estimates of the targets. As a result, substantially smaller number of hypothesis tracks will be created during track-to-estimate association, requiring a simpler hypotheses pruning scheme in the PHD-with-association filter. Similarly, the MHT-with-PHD filter also benefits from the almost clutter free estimates of targets that are used to eliminate unlikely measurements before applying MHT. The performance of both of these schemes mainly depends on the accuracy of the PHD estimates of the targets and hence on the details of the particle approximation of the PHD filter; i.e. the number of particles per target, the choice of the proposal density function and the re-sampling scheme. It also depends on the accuracy of the peak extracting methods employed to extract peaks from the PHD function at each time step.

The peak-to-track association technique presented in [17] considers the formation of tracks based on association of peaks over two consecutive time steps by keeping validation gates on each of the tracks. It does not use estimates that will arrive later in time to resolve the uncertainty in the association. The algorithm presented in [17] is primarily intended for tracking scenarios with linear/Gaussian state models. Moreover, the paper proposes to sample particles around each measurement as well as proposes to modify the update step of the SMC-PHD filter so that the contribution of the measurements that fall within the validation gates of existing tracks are weighted more than the ones that fall outside the validation gates. Drawing samples around every measurement means that the computational cost of the algorithm now depends on the number of measurements. This will also results in increased number of false tracks being picked up. Moreover, the effect this proposed modification may have on the convergence properties of the SMC-PHD filter have not been looked into. Recent work in [18] proposes to find the best association of target estimates over consecutive time steps and is a special case of the PHD-with-association filter in which the uncertainty in the association are not improved by using the estimates that will arrive later in time.

It has been claimed in [17] that the proposed PHD filter based scheme outperforms a data association algorithm like MHT. It is misleading to claim the particle-PhD filter outperforms the MHT or vice versa, as the performances of both schemes depend on their particular implementations and how much computational load they are prepared to tolerate. The performance of the MHT can be improved upon by allowing it consider more exhaustive associations amongst measurements over time. Similarly, the performance of the particle-PHD filter can be improved by using larger number of particles per target, and by employing better proposal density functions and better peak extraction methods. Another claim in [17] is that the proposed SMC-PHD based scheme is computationally worse off than the MHT. This might be true for tracking scenarios that consist of linear/Gaussian state models. However, for general tracking purposes with nonlinear/non-Gaussian state models, multiple particle filters (working in parallel) would be required to operate on each track hypothesis and would make the MHT computationally more expensive than the PHD filter. The comparison of the computational cost between a SMC-PHD based multi-target tracker and the MHT requires careful considerations of implementation details of both schemes and their particular applications.

The PHD-with-association filter proposed in this paper is applicable to tracking scenarios with non-linear/non-Gaussian state models. The subsequent estimates given by the PHD filter have been used to reduce the uncertainty in the track-to-estimate association over time. The computational cost of the SMC-PHD filter does not dependent on the number of measurements.

V. Conclusion

This paper presents novel schemes for extending the use of the SMC-PHD filter in order to obtain track-valued estimates of individual targets in multi-target tracking. Simulation results have been presented for tracking multiple targets for the proposed schemes. It is shown that the track-valued estimates of the PHD-with-association filter are almost free of false tracks. The ‘track-to-estimate association’ functionality eliminates some of the spurious clutter initially picked by the SMC-PHD filter. The Wasserstein distance is also used to measure the multi-target miss distance of the point state estimates given by the PHD filter. It has been shown that the PHD-with-association filter performs comparably well against the track-oriented MHT that has been used a benchmark algorithm. It should be noted that the performance of MHT is subject to the particular implementation and can be improved upon at the expense of added computation cost and complexity. However, when the non-linearity and/or non-Gaussianity in the target dynamics and/or measurement process are considerable, the PHD-with-association filter should be preferred over MHT as the EKF used would not perform satisfactorily and would require running a large number of particle filters on each hypothesis track. Furthermore, we show that the PHD filter can be used as a clutter filter in order to improve the performance of the MHT, however at the cost of added computational load.

The performance of the schemes proposed above mainly depends on the accuracy of the point state estimates given by the SMC-PHD filter. As a result, further extensions to this work may consider the improvement in the performance of the SMC-PHD filter via better proposal densities, resampling schemes and more importantly better techniques for peak extraction. Further study is needed to study the performance of the MHT-with-PHD clutter filter and the merit of the tradeoff between the added computational cost of global gating using the PHD filter and the improvement in the computational cost of the MHT due to global gating. Various issues associated with the complexities and computational requirements of the proposed schemes and optimal methods for initiating tracks with the PHD filter are beyond the scope of this paper and are subjects of further research.

ACKNOWLEDGEMENTS

The first author would like to extend sincere gratitude to Dr. Mahendra Mallick for his guidance and suggestions for the implementation of the track-oriented MHT algorithm.

This work was supported by a large Australian Research Council (ARC) grant and Cooperative Research Centre for Sensor Signal and Information Processing (CSSIP), Department of Electrical & Electronic Engineering, The University of Melbourne, Parkville, Victoria 3010, Australia.

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