Multi-Target Tracking with Time-Varying Clutter Rate and Detection Profile: Application to Time-lapse Cell Microscopy Sequences

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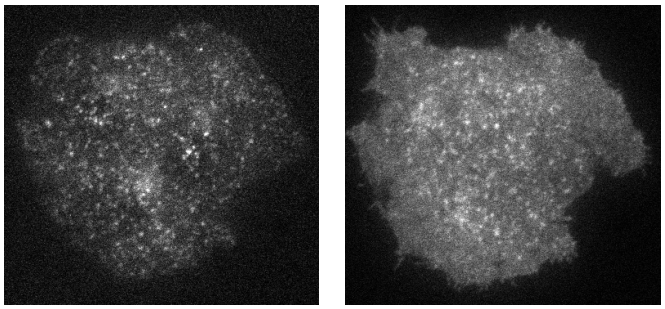
***Abstract*—Quantitative analysis of the dynamics of tiny cellular and sub-cellular structures, known as particles, in time-lapse cell microscopy sequences requires the development of a reliable multi-target tracking method capable of tracking numerous similar targets in the presence of high levels of noise, high target density, complex motion patterns and intricate interactions. In this paper, we propose a framework for tracking these structures based on the random finite set Bayesian filtering framework. We focus on challenging biological applications where image characteristics such as noise and background intensity change during the acquisition process. Under these conditions, detection methods usually fail to detect all particles and are often followed by missed detections and many spurious measurements with unknown and time-varying rates. To deal with this, we propose a bootstrap filter composed of an estimator and a tracker. The estimator adaptively estimates the required meta parameters for the tracker such as clutter rate and the detection probability of the targets, while the tracker estimates the state of the targets. Our results show that the proposed approach can outperform state-of-the-art particle trackers on both synthetic and real data in this regime.**

***Index Terms*—Multi-target tracking, particle tracking, fluorescence microscopy, Bayesian estimation, random finite set, CPHD, clutter rate, detection probability.**

I. Introduction

THE ability to accurately monitor cellular and sub-cellular structures in their native biological environment has enormous potential in addressing open questions in cell biology. In various applications, one of the key steps for understanding biological phenomena is to assess the motion of these structures. Recent developments in time-lapse cell microscopy imaging systems have had a great impact on the analysis of these dynamics. However, visual inspection of sequences acquired by these imaging techniques requires manual tracking of many tiny structures in numerous noisy images. Thus, automated multi-target tracking methods have been extensively used in different biological applications in the last decade [1–25].



(a) (b)

Fig. 1. Two images of a TIRFM sequence visualizing fluorescently tagged vesicles (bright spots) close to the plasma membrane of a pancreatic beta cell (a) before and (b) after injection of insulin. Clearly, the background intensity and noise level noticeably increase during the acquisition.

Despite significant technical advances made in automatically tracking moving objects, tracking microscopic structures, known as particles [26], remains a challenging task due to the complex nature of biological sequences. The microscopic sequences are usually populated with similar tiny structures having intricate motion patterns and sophisticated interactions with other structures. Moreover, the structures may enter or disappear from the field of view or be occluded by other objects. In addition in some imaging techniques, i.e. fluorescence microscopy imaging, the sequences are contaminated with a high degree of noise. Under these conditions, detection methods usually fail to detect all particles and generate many spurious measurements (clutter) [14, 27]. To be successfully applied in many biological applications, multi-target tracking methods should be able to track an unknown and time-varying number of similar particles in the presence of clutter and detection uncertainty.

To this end, many particle tracking approaches have been proposed in literature [1–24]. Some of the most popular particle tracking approaches are based on detection followed by a deterministic linking procedure. Here, each particle is separately detected in each frame. Then, a deterministic solution, e.g. an optimization technique, links the corresponding targets between frames [3–7]. The performance of these algorithms is often sensitive to the detection algorithm and may degrade in

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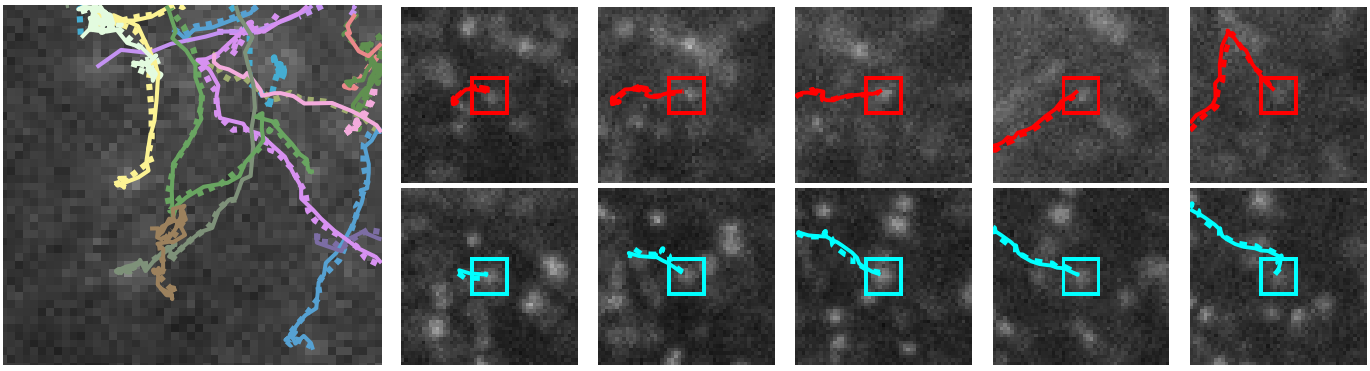

(a) (b)

Fig. 9. Some examples of tracked vesicles using the proposed bootstrap filter (dashed line) against the ground truth (solid line) in the synthetic and real TIRFM sequences. (a) The tracking results of multiple crossing particles with maneuvering motions in the synthetic TIRFM sequences. (b) The resulting trajectory of two faint vesicles from the synthetic (Top row) and real (Bottom row) TIRFM sequences in different time frames. For an improved visualization, the other tracks are eliminated. These particles move in different background intensity and noise level.

Gaussian system models, the sequential Monte Carlo (SMC) implementation of the filters is required. This implementation will also allows us to further evaluate the performance of our bootstrap filter and compare it against the SMC-PHD [41], SMC-CPHD [31] and the traditional particle [34] filters.

APPENDIX

For completeness, we now derive the recursive equations for both the multiple model CPHD and λ-p_D-CPHD filters in this section. The following notations are used in throughout this Appendix. The binomial and permutation coefficients are denoted by C_j^l and P_j^n, respectively. $\langle \cdot, \cdot \rangle$ is the inner product operation between two continuous or two discrete functions. The elementary symmetric function of order j defined for a finite set Z of real numbers is denoted by

$$e_j(Z) = \sum_{S \subseteq Z, |S|=j} \left(\prod_{\theta \in S} \theta \right), \tag{29}$$

where $|S|$ is the cardinality of a set S and $e_0(Z) = 1$ [42].

TABLE III
THE AVERAGED LOCATION, CARDINALITY, OSPA AND OSPA-T ERRORS OF THE TRACKERS

in pixel with the metric parameters $p = 1$ and $c = \ell = 10$ (pixel) for a real image sequence.

Method	Location	Cardinality	OSPA	OSPA-T
B-MM-CPHD	3.91	**1.04**	**4.95**	**6.30**
MM-λ-p_D-CPHD	4.01	1.23	5.24	6.56
MM-CPHD	3.92	2.07	5.99	7.23
MM-PHD [16]	4.05	2.23	6.28	7.64
IMM-JPDA [19]	0.93	5.75	6.68	7.25
MHT [14]	1.21	4.28	5.49	6.33
P-Tracker [6]	**0.55**	5.45	6.00	7.32
U-Tracker [4]	1.96	3.63	5.59	6.44

A. Multiple Model CPHD Recursions

Prediction step: Suppose at time $k-1$, the posterior cardinality distribution ρ_{k-1} and posterior intensity $\underline{v}_{k-1}$ are known. The predicted cardinality distribution $\rho_{k|k-1}$ and predicted intensity $\underline{v}_{k|k-1}$ are calculated by

$$\rho_{k|k-1}(n) = \sum_{j=0}^{n} \rho_\Gamma(n-j) \Pi[\underline{v}_{k-1}, \rho_{k-1}](j), \tag{30}$$

$$\underline{v}_{k|k-1}(x,r) = \gamma(x)\pi(r) + \sum_{\acute{r}} \int \underline{p}_S(\acute{x},\acute{r}) f(x|\acute{x},r) \tau(r|\acute{r}) \underline{v}_{k-1}(\acute{x},\acute{r}) d\acute{x}, \tag{31}$$

where $\Pi[\underline{v}_{k-1}, \rho_{k-1}](j) =$

$$\sum_{l=j}^{\infty} C_j^l \rho_{k-1}(l) \frac{\langle \underline{p}_S, \underline{v}_{k-1} \rangle^j \langle 1 - \underline{p}_S, \underline{v}_{k-1} \rangle^{l-j}}{\langle 1, \underline{v}_{k-1} \rangle^{l-j}}. \tag{32}$$

Note that in the multiple model approach, the inner product function operates on both the kinematic state and the model, i.e. $\langle \underline{p}_S, \underline{v}_{k-1} \rangle = \sum_{\acute{r}} \int p_S(\acute{x},\acute{r}) \underline{v}_{k-1}(\acute{x},\acute{r}) d\acute{x}$.

Update step: If at time k, the predicted cardinality distribution $\rho_{k|k-1}$, predicted intensity $\underline{v}_{k|k-1}$ and set of measurement Z_k are given, the updated cardinality distribution ρ_k and updated intensity $\underline{v}_k$ are calculated by

$$\rho_k(n) = \frac{\underline{\Upsilon}^0 \left[\underline{v}_{k|k-1}, Z_k \right](n) \rho_{k|k-1}(n)}{\left\langle \underline{\Upsilon}^0 \left[\underline{v}_{k|k-1}, Z_k \right], \rho_{k|k-1} \right\rangle}, \tag{33}$$

$$\underline{v}_k(x,r) = \underline{v}_{k|k-1}(x,r) \times \left[(1 - \underline{p}_D(x,r)) \frac{\left\langle \underline{\Upsilon}^1 \left[\underline{v}_{k|k-1}, Z_k \right], \rho_{k|k-1} \right\rangle}{\left\langle \underline{\Upsilon}^0 \left[\underline{v}_{k|k-1}, Z_k \right], \rho_{k|k-1} \right\rangle} + \sum_{z \in Z_k} \underline{\psi}_z(x,r) \frac{\left\langle \underline{\Upsilon}^1 \left[\underline{v}_{k|k-1}, Z_k \backslash z \right], \rho_{k|k-1} \right\rangle}{\left\langle \underline{\Upsilon}^0 \left[\underline{v}_{k|k-1}, Z_k \right], \rho_{k|k-1} \right\rangle} \right], \tag{34}$$

where $\underline{\Upsilon}^{u}\left[\underline{v}_{k|k-1}, Z_k\right](n) =$

$$\sum_{j=0}^{\min(|Z_k|,n)} (|Z_k|-j)!\rho_K(|Z_k|-j)P^{n}_{j+u} \times \frac{\langle 1-\underline{p}_D, \underline{v}_{k|k-1}\rangle^{n-(j+u)}}{\langle 1, \underline{v}_{k|k-1}\rangle^{n}} e_j\left(\Xi(\underline{v}_{k|k-1}, Z_k)\right) \tag{35}$$

where

$$\Xi(\underline{v}_{k|k-1}, Z_k) = \left\{\left\langle \underline{v}_{k|k-1}, \underline{\psi}_z\right\rangle : z \in Z_k\right\}, \tag{36}$$

and

$$\underline{\psi}_z(x,r) = \frac{\langle 1, \kappa\rangle}{\kappa(z)}\underline{g}(z|x,r)\underline{p}_D(x,r). \tag{37}$$

B. Multiple Model λ-p_D-CPHD Recursions

Prediction step: Suppose at time $k-1$, the hybrid cardinality distributions $\ddot{\rho}_{k-1}$ and the posterior intensity distribution for actual targets $\underline{v}^{(1)}_{k-1}$ and clutter generators $\underline{v}^{(0)}_{k-1}$ are given. The hybrid predicted cardinality distribution $\ddot{\rho}_{k|k-1}$ and predicted intensity for actual targets $\underline{v}^{(1)}_{k|k-1}$ and clutter generators $\underline{v}^{(0)}_{k|k-1}$ are calculated by

$$\ddot{\rho}_{k|k-1}(\ddot{n}) = \sum_{j=0}^{\ddot{n}} \ddot{\rho}_{\Gamma}(\ddot{n}-j)\sum_{l=j}^{\infty} C^{l}_{j}\ddot{\rho}_{k-1}(l)(1-\phi)^{l-j}\phi^{j} \tag{38}$$

where

$$\phi = \left(\frac{\langle \underline{p}^{(1)}_S, \underline{v}^{(1)}_{k-1}\rangle + \langle \underline{p}^{(0)}_S, \underline{v}^{(0)}_{k-1}\rangle}{\langle 1, \underline{v}^{(1)}_{k-1}\rangle + \langle 1, \underline{v}^{(0)}_{k-1}\rangle}\right), \tag{39}$$

$$\underline{v}^{(1)}_{k|k-1}(x,a,r) = \underline{\gamma}^{(1)}_{k}(x,a)\pi(r) + \sum_{\acute{r}}\int\int_0^1 \left[p^{(1)}_S(\acute{x},\acute{r})f^{(1)}(x|\acute{x},r)\times f^{(\Delta)}(a|\acute{a},\acute{r})\tau(r|\acute{r})\underline{v}^{(1)}_{k-1}(\acute{x},\acute{a},\acute{r})d\acute{x}d\acute{a}\right], \tag{40}$$

$$\underline{v}^{(0)}_{k|k-1}(b) = \underline{\gamma}^{(0)}(b) + p^{(0)}_S\underline{v}^{(0)}_{k-1}(b). \tag{41}$$

Update step: If at time k, the predicted intensity for actual targets $\underline{v}^{(1)}_{k|k-1}$, the predicted intensity for clutter generators $\underline{v}^{(0)}_{k|k-1}$, the predicted hybrid cardinality distribution $\ddot{\rho}_{k|k-1}$ and set of measurement Z_k are all given and the function $\ddot{\underline{\Upsilon}}^{u}\left[\ddot{\underline{v}}_{k|k-1}, Z_k\right](\ddot{n})$ defined as follows:

$$\ddot{\underline{\Upsilon}}^{u}\left[\ddot{\underline{v}}_{k|k-1}, Z_k\right](\ddot{n}) = \begin{cases} 0 & \ddot{n} < |Z_k|+u \\ P^{\ddot{n}}_{|Z_k|+u}\Phi^{\ddot{n}-(|Z_k|+u)} & \ddot{n} \geq |Z_k|+u \end{cases} \tag{42}$$

where

$$\Phi = 1 - \frac{\left\langle \underline{v}^{(1)}_{k|k-1}, \underline{p}^{(1)}_D\right\rangle + \left\langle \underline{v}^{(0)}_{k|k-1}, \underline{p}^{(0)}_D\right\rangle}{\left\langle 1, \underline{v}^{(1)}_{k|k-1}\right\rangle + \left\langle 1, \underline{v}^{(0)}_{k|k-1}\right\rangle}, \tag{43}$$

where $\underline{p}^{(1)}_D(x,a,r) = a$ and $\underline{p}^{(0)}_D(b) = b$.

The updated cardinality distribution $\ddot{\rho}_k$ and the updated intensity distribution for actual targets $\underline{v}^{(1)}_k$ and clutter generators $\underline{v}^{(0)}_k$ are given as follows

$$\ddot{\rho}_k(\ddot{n}) = \begin{cases} 0 & \ddot{n} < |Z_k|, \\ \dfrac{\ddot{\underline{\Upsilon}}^{0}\left[\ddot{\underline{v}}_{k|k-1}, Z_k\right](\ddot{n})\ddot{\rho}_{k|k-1}(\ddot{n})}{\left\langle \ddot{\underline{\Upsilon}}^{0}, \ddot{\rho}_{k|k-1}\right\rangle} & \ddot{n} \geq |Z_k|, \end{cases} \tag{44}$$

$$\underline{v}^{(1)}_k(x,a,r) = \underline{v}^{(1)}_{k|k-1}(x,a,r)\left[\frac{(1-a)\frac{\left\langle \ddot{\underline{\Upsilon}}^{1}\left[\ddot{\underline{v}}_{k|k-1}, Z_k\right], \ddot{\rho}_{k|k-1}\right\rangle}{\left\langle \ddot{\underline{\Upsilon}}^{0}\left[\ddot{\underline{v}}_{k|k-1}, Z_k\right], \ddot{\rho}_{k|k-1}\right\rangle}}{\left\langle 1, \underline{v}^{(1)}_{k|k-1}\right\rangle + \left\langle 1, \underline{v}^{(0)}_{k|k-1}\right\rangle} + \sum_{z\in Z_k}\frac{a.g(z|x,r)}{\left\langle \underline{v}^{(0)}_{k|k-1}, p^{(0)}_D\mathcal{K}\right\rangle + \left\langle \underline{v}^{(1)}_{k|k-1}, p^{(1)}_D g(z|\cdot)\right\rangle}\right], \tag{45}$$

$$\underline{v}^{(0)}_k(b) = \underline{v}^{(0)}_{k|k-1}(b)\left[\frac{(1-b)\frac{\left\langle \ddot{\underline{\Upsilon}}^{1}\left[\ddot{\underline{v}}_{k|k-1}, Z_k\right], \ddot{\rho}_{k|k-1}\right\rangle}{\left\langle \ddot{\underline{\Upsilon}}^{0}\left[\ddot{\underline{v}}_{k|k-1}, Z_k\right], \ddot{\rho}_{k|k-1}\right\rangle}}{\left\langle 1, \underline{v}^{(1)}_{k|k-1}\right\rangle + \left\langle 1, \underline{v}^{(0)}_{k|k-1}\right\rangle} + \sum_{z\in Z_k}\frac{b.\mathcal{K}(z)}{\left\langle \underline{v}^{(0)}_{k|k-1}, p^{(0)}_D\mathcal{K}\right\rangle + \left\langle \underline{v}^{(1)}_{k|k-1}, p^{(1)}_D g(z|\cdot)\right\rangle}\right]. \tag{46}$$

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