JOINT DETECTION AND TRACKING OF MULTIPLE MANEUVERING TARGETS IN CLUTTER USING RANDOM FINITE SETS

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ABSTRACT

Joint multi-target detection and tracking is a challenging problem due to several factors such as the number of targets being an unknown time-varying random parameter, and the target generated measurements being obscured by clutter. The theory of random finite sets permits an elegant formulation of the joint mult-itarget detection and tracking problem in a Bayesian framework. In particular, the random finite set formalism leads to the probability hypothesis density (PHD) filtering method, which realizes Bayesian joint detection and tracking in a suboptimal but numerically tractable manner. In this paper the PHD filter is applied to jointly detect and track multiple maneuvering targets. The tracking performance fidelity of the PHD filter is demonstrated by numerical results.

Keywords: Multi-target Tracking, Optimal Filtering, Particle Methods, Point Processes, Random Sets, Sequential Monte Carlo.

1. INTRODUCTION

Non-maneuvering target motion can be described by a fixed model. On the other hand, for maneuvering targets, the motion model changes with time. The large uncertainty in the system model is reflected by the rich variety of maneuvers that a target can exhibit. In general maneuvers are modeled as a random proccess with some known characteristics [8].

This paper considers the problem of jointly detecting and tracking multiple maneuvering targets in clutter. Unlike multi-target tracking problems where the number of targets is known, in joint detection and tracking, new targets can appear and old targets can die from time to time, i.e. the number of targets varies with time in a random fashion. Each existing target may or may not generate a measurement, in addition, the sensor also receives a set spurious measurements not originated from any targets. Thus, this problem involves jointly estimating the number of targets and their states from a set of measurements of uncertain origin.

One approach to tracking multiple targets is to apply single target tracking techniques to each target. However, this would require correct association of measurements to targets [2]. The most likely association of measurements to targets at each time step can be used. This only works well for widely spaced targets, low measurement noise and low clutter, in which case, the most likely hypothesis would stand out from the rest. The Joint Probabilistic Data Association (JPDA) method caters for uncertain associations by allowing a track to be updated by a weighted sum of all observations in its gate [1, 2]. However, JPDA can only handle a fixed number of targets. Multiple Hypothesis Tracking (MHT) propagates a number of possible association hypotheses in time so that association uncertainty can be reduced as new data arrive [2, 10]. However, the number of hypotheses grow exponentially over time, thus rendering MHT computationally infeasible. Ad-hoc methods to reduce the number of hypotheses are required for practical implementations.

An association-free formulation can be achieved by treating the collection of targets as a single *set-valued state*, and the collection of observations as a single *set-valued observation*. By modeling set-valued states and set valued observations as random finite sets, the multi-target joint detection and tracking problem can be cast as a Bayesian multi-object filtering problem [9, 14, 15]. While the Bayes multi-target filter is computationally intractable, an approximation known as the Probability Hypothesis Density (PHD) filter offers a cheaper alternative [9]. The PHD filter is a recursion propagating the posterior intensity function (or PHD) of the multi-target state. This propagation involves multiple integrals that have no closed form expressions in general. Recently, this problem was alleviated by using a Sequential Monte Carlo (SMC) implementation known as the particle-PHD (or SMC-PHD) filter [13, 15] (see also [11] and [16] for special cases of the particle-PHD filter). Furthermore, convergence analysis in [15] indicated that the proposed SMC implementation can provide good approximations to the PHD recursion given a sufficient number of particles.

This paper applies the PHD filter to solve the problem of joint detection and tracking of multiple maneuvering targets with cluttered measurements. It will be demonstrated numerically in Section 4 that the PHD filter not only provides an accurate estimation of the number of targets (which is time-varying), it also reliably tracks the target motions.

2. DATA MODEL

2.1. Single Maneuvering Target without Clutter

Let ξ_k denote the kinematic state vector of a target at time k (e.g., the target coordinate and velocity). The dynamics of the target states can be modeled by a state space equation

$$\xi_k = F\xi_{k-1} + Eu_{k-1} + G\eta_{k-1}, \tag{1}$$

where F, E and G are given matrices, the vector u_k is a control input that accounts for the maneuvering behaviour of the target, and η_k is the process noise. Very often u_k is unknown to the tracking system and it is common (if not standard) to treat u_k as if it were "noise" [8].

The maneuvering dynamics in (1) may be recast to an autonomous form

$$x_k = \mathbf{\Phi}_k(x_{k-1}, w_{k-1}), \tag{2}$$

with augmented state vector x_k, process noise w_k, and nonlinear state transition $\mathbf{\Phi}_k(.,.)$. For instance, we can have $x_k = [\xi_k^T, u_k^T]^T$ and use w_{k-1} to drive the random (Markov) time variations of both ξ_k and u_k. Alternatively, we can simply have $x_k = \xi_k$ and use w_{k-1} to model both the process noise and control input. More detailed descriptions for the stochastic modeling of maneuvers can be found in [3, 8].

In a radar tracking system, the target measurement consists of the range and bearing of the target. The measurement at time k can be modeled by

$$z_k = \begin{bmatrix} r(x_k) \\ \theta(x_k) \end{bmatrix} + v_k, \tag{3}$$

where $r(x_k)$ and $\theta(x_k)$ are, respectively, the range and bearing of the given state x_k, and v_k is the measurement noise.

In Bayesian estimation, the state and measurement equations (2) and (3) can be, respectively, captured by the state transition probability density and likelihood function

$$f_{k|k-1}(x_k|x_{k-1}), \tag{4}$$

$$g_k(z_k|x_k). \tag{5}$$

Let $z_{1:k} = (z_1, \ldots, z_k)$, and denote the posterior density by

$$p_{k|k}(x_k|z_{1:k}). \tag{6}$$

The posterior density encapsulates all information about the state, and can be computed using the Bayes recursion

$$\begin{aligned} & p_{k|k-1}(x_k|z_{1:k-1}) \\ &= \int f_{k|k-1}(x_k|x)p_{k-1|k-1}(x|z_{1:k-1})\lambda(dx), \end{aligned} \tag{7}$$

$$\begin{aligned} & p_{k|k}(x_k|z_{1:k}) \\ &= \frac{g_k(z_k|x_k)p_{k|k-1}(x_k|z_{1:k-1})}{\int g_k(z_k|x)p_{k|k-1}(x|z_{1:k-1})\lambda(dx)}. \end{aligned} \tag{8}$$

given the initial density $p_{0|0}(x_0)$. From the posterior density at time k, we can estimate x_k using either the minimum mean squared error (MMSE) criterion or the maximum *a posteriori* (MAP) criterion.

2.2. Multiple Maneuvering Targets in Clutter

In a multi-target scenario, each (maneuvering) target moves independently according to the dynamic model (1). New targets can appear and existing targets can disappear randomly. At time k, let $M(k)$ be the number of targets present with states $x_{k,1}, \ldots, x_{k,M(k)}$. Each of these targets continues to exist at the next time step with certain probability. In addition new targets can come into the scene with certain probability.

Each target generates an observation with probability $p_D(x_{k,i})$ or no observation with probability $p_M(x_{k,i}) = 1 - p_D(x_{k,i})$, $i = 1, ...M(k)$. In addition, the sensor also receives a set of false alarms or clutter, i.e. spurious measurements not originated from any targets. Thus at time k, we have $N(k)$ measurements with values $z_{k,1}, \ldots, z_{k,N(k)}$, which consists of clutter and target originated measurements. Clutter are considered to be statistically independent from the target originated measurement. The number of clutter points is often modeled as a Poisson random variable with mean λ_k and each clutter point is independently distributed according to the probability density $c(z)$. A typical choice for the clutter probability density is the uniform density on the observation space.

The problem of joint detection and tracking of multiple targets is difficult because the number of targets $M(k)$, the number of measurements $N(k)$ both vary randomly in time and it is not known which target generated which measurement. In the next section, our multi-target detection and tracking approach using the random finite set concepts will be presented.

3. RANDOM FINITE SET FORMULATION

Let E_s and E_o denote the state and observation spaces respectively. The key in the random finite set formulation is to treat the target set and measurement set

$$X_k = \{x_{k,1}, \ldots, x_{k,M(k)}\} \subset E_s, \tag{9}$$

$$Z_k = \{z_{k,1}, \ldots, z_{k,N(k)}\} \subset E_o \tag{10}$$

as the *multi-target state* and *multi-target measurement* respectively. Note that the cardinalities of X_k and Z_k are allowed to be time variant. In the first subsection, we describe the statistical modeling of the multi-target state and measurement. In the second subsection, the PHD filtering technique for solving the multi-target set estimation problem is considered.

3.1. The Multi-target Bayes filter

Analogous to single target systems, where uncertainty is characterised by modeling the states and measurements by random vectors, uncertainty in a multi-target system is characterised by modeling multi-target states X_k and multi-target measurements Z_k as random finite sets (RFS) Ξ_k and Σ_k on the (single-target) state and observation spaces E_s and E_o respectively (for details on RFS see [4, 5, 9, 12, 15]). Given a realisation X_{k-1} of Ξ_{k-1}, the multi-target state at time k can be modeled by

$$\Xi_k = S_k(X_{k-1}) \cup B_k(X_{k-1}) \cup \Gamma_k, \tag{11}$$

where

$S_k(X_{k-1})$ = RFS of targets that have survived at time k,

$B_k(X_{k-1})$ = RFS of targets spawned from X_{k-1},

Γ_k = RFS of targets that spontaneously appeared at time k.

The statistical behaviour of the RFS Ξ_k is characterised by the multi-target transition density $f_{k|k-1}(X_k|X_{k-1})$ in an analogous fashion to the Markov transition density for single target. Treatment of densities of RFS can be found in [4, 5, 9, 12, 15].

Similarly, given a realisation X_k of Ξ_k, the multi-target measurement can be modeled by the RFS

$$\Sigma_k = \Theta_k(X_k) \cup K_k(X_k), \tag{12}$$

where

$\Theta_k(X_k)$ = RFS of target generated measurements,

$K_k(X_k)$ = RFS of clutter.

The statistical behaviour of the RFS Σ_k is described by the multi-target likelihood $g_k(Z_k|X_k)$.

Let $p_{k|k}(X_k|Z_{1:k})$ denote the multi-target posterior density. Then, analogous to the single target case, the optimal multi-target Bayes filter is given by the recursion

$$\begin{aligned} & p_{k|k-1}(X_k|Z_{1:k-1}) \\ &= \int f_{k|k-1}(X_k|X) p_{k-1|k-1}(X|Z_{1:k-1}) \mu_s(dX), \end{aligned} \tag{13}$$

$$\begin{aligned} & p_{k|k}(X_k|Z_{1:k}) \\ &= \frac{g_k(Z_k|X_k) p_{k|k-1}(X_k|Z_{1:k-1})}{\int g_k(Z_k|X) p_{k|k-1}(X|Z_{1:k-1}) \mu_s(dX)}. \end{aligned} \tag{14}$$

where μ_s is the extended Lebesque measure on the space of finite subsets of E_s [14, 15].

The recursion (13-14) involves the evaluation of multiple integrals on the space of finite sets and thus, the computational intractability is far more severe than its single-target counterpart [15]. A generic SMC implementation has proposed in [15] but it is still computationally demanding. A computationally cheaper alternative can be obtained by approximating the multi-target densities of interest using their 1st order moments.

3.2. The Probability Hypothesis Density (PHD) filter

The 1st momment or intensity measure V_Ξ of a RFS Ξ on R^n with probability distribution P_Ξ is defined by

$$V_\Xi(S) \equiv \mathbf{E}[|\Xi \cap S|] = \int |X \cap S| \, P_\Xi(dX) \tag{15}$$

for each subset[1] S of R^n, where $|X|$ denotes the cardinality of X. The density v_Ξ of the intensity V_Ξ w.r.t. the Lebesque measure (if one exists) is called the intensity function, also known in the tracking literature as the Probability Hypothesis Density (PHD). Note that the integral

$$\int_S v_\Xi(x) \lambda(dx) \tag{16}$$

gives the expected number of elements of Ξ that are in the region S.

For the RFS in (11), the expected number of targets at time k in a region S is $\int_S v_{\Xi_k}(x_k)\lambda(dx_k)$. The peaks of the intensity function v_{Ξ_k} can be used to obtain estimates for the target states at time k.

In the PHD filter, we are interested in the intensity functions $v_{k|k}(x)$ and $v_{k|k-1}(x)$ that correspond to the multi-target densities $p_{k|k-1}(X_k|Z_{1:k-1})$ and $p_{k|k}(X_k|Z_{1:k})$ in (13-14), respectively. Let

γ_k = intensity function of the birth RFS Γ_k,

$\beta_{k|k-1}(.|\zeta)$ = intensity function of the RFS $B_k(\{\zeta\})$ spawned by a target with previous state ζ,

$e_{k|k-1}(\zeta)$ = probability that the target still exist at time k given that its previous state is ζ,

κ_k = intensity function of the clutter RFS $K_k(X_k)$.

Under certain mild assumptions, it can be shown that [9]

$$\begin{aligned} & v_{k|k-1}(x) \\ &= \int \phi_{k|k-1}(x,\zeta) v_{k-1|k-1}(\zeta) \lambda(d\zeta) + \gamma_k(x), \end{aligned} \tag{17}$$

$$\begin{aligned} & v_{k|k}(x) \\ &= \left[p_M(x) + \sum_{z \in Z_k} \frac{\psi_{k,z}(x)}{\kappa_k(z) + \langle \psi_{k,z}, v_{k|k-1} \rangle} \right] v_{k|k-1}(x), \end{aligned} \tag{18}$$

[1] Subsets are assumed Borel measureable where appropriate.

where

$$\begin{aligned}\phi_{k|k-1}(x,\xi) &= e_{k|k-1}(\xi)f_{k|k-1}(x|\,\xi)+\beta_{k|k-1}(x|\,\xi),\\ \psi_{k,z}(x) &= p_D(x)g_k(z|\,x),\\ \langle f,g\rangle &= \int f(x)g(x)\lambda(dx).\end{aligned}$$

The approximation in (18) is based on the premise that the predicted multi-target density $p_{k|k-1}$ is Poisson. (These assumptions are justifiable when the measurement noise and the false alarm rate are small.)

Like the multi-target Bayes recursion in (13-14), the propagation of the intensity functions in (17-18) are recursive in nature. Since intensity functions are defined on a single-target state space, their propagation is much less complex than that of the multi-target Bayes recursion. However, Eqns. (17-18) consist of multiple integrals that have no closed form expressions in general. Recently, this problem has been solved by a Sequential Monte Carlo (SMC) implementation of the PHD recursion, known as the particle-PHD filter [13], [15]. The particle-PHD filter is summarized as follows (more details can be found in [15]).

Particle-PHD filter

At time $k \geq 1$,

Step 1: *Prediction*

- Sample

$$\widetilde{x}_k^{(i)} \sim \begin{cases} q_k\left(\cdot|\,x_{k-1}^{(i)},Z_k\right),\ i=1,...,L_{k-1} \\ p_k\left(\cdot|\,Z_k\right),\ i=L_{k-1}+1,...,L_{k-1}+J_k\end{cases}$$

and compute the predicted weights

$$\widetilde{w}_{k|k-1}^{(i)} = \begin{cases} \frac{\phi_k\left(\widetilde{x}_k^{(i)},x_{k-1}^{(i)}\right)w_{k-1}^{(i)}}{q_k\left(\widetilde{x}_k^{(i)}\middle|x_{k-1}^{(i)},Z_k\right)},\ i=1,...,L_{k-1} \\ \frac{\gamma_k\left(\widetilde{x}_k^{(i)}\right)}{J_k p_k\left(\widetilde{x}_k^{(i)}\middle|Z_k\right)},\ i=L_{k-1}+1,...,L_{k-1}+J_k\end{cases}$$

Step 2: *Update*

- For each $z \in Z_k$, compute

$$C_k(z) = \sum_{j=1}^{L_{k-1}+J_k} \psi_{k,z}(\widetilde{x}_k^{(j)})\widetilde{w}_{k|k-1}^{(j)}.$$

- For $i=1,...,L_{k-1}+J_k$, update weights

$$\widetilde{w}_k^{(i)} = \left[\upsilon(\widetilde{x}_k^{(i)}) + \sum_{z\in Z_k}\frac{\psi_{k,z}(\widetilde{x}_k^{(i)})}{\kappa_k(z)+C_k(z)}\right]\widetilde{w}_{k|k-1}^{(i)}.$$

Step 3: *Resampling*

- Compute the total mass $\widehat{N}_{k|k} = \sum_{j=1}^{L_{k-1}+J_k}\widetilde{w}_k^{(j)}$,
- Resample $\left\{\frac{\widetilde{w}_k^{(i)}}{\widehat{N}_{k|k}},\widetilde{x}_k^{(i)}\right\}_{i=1}^{L_{k-1}+J_k}$ to get $\left\{\frac{w_k^{(i)}}{\widehat{N}_{k|k}},x_k^{(i)}\right\}_{i=1}^{L_k}$,
- Rescale the weights by $\widehat{N}_{k|k}$ to get $\left\{w_k^{(i)},x_k^{(i)}\right\}_{i=1}^{L_k}$.

The idea behind the particle-PHD filter is to use particles to approximate the PHDs recursively. To estimate the number of targets from the PHD particles, we can use

$$\begin{aligned}\hat{N}_k &= \text{round}\left(\int_{E_s} v_{k|k}(x)dx\right)\\ &\approx \text{round}(\hat{N}_{k|k})\end{aligned} \tag{19}$$

where $\hat{N}_{k|k}$ is defined in Step 3 of the particle-PHD filter. To estimate the multiple target states, a clustering algorithm, such as the K-means clustering algorithm [6], is used to determine the center points of clusters in the particle approximation of the posterior intensity function. These center points can then be used as estimates of individual target states.

4. SIMULATIONS

4.1. Example

We model the maneuvering target dynamics by the constant turn model with unknown turn rate [8]. In this model a state vector consists of the kinematic components and the turn rate i.e. $x_k = [\ p_{x,k}, v_{x,k}, p_{y,k}, v_{y,k}, \omega_k\ ]^T$, where $(p_{x,k}, p_{y,k})$, $(v_{x,k}, v_{y,k})$, and ω_k are, respectively, the coordinate, velocity, and turn rate of a target at time k. Given a sampling period T , the state space equation is given by

$$x_k = \begin{bmatrix} F(e_5^T x_{k-1}) & 0 \\ 0 & 1\end{bmatrix} x_{k-1} + \begin{bmatrix} G & 0 \\ 0 & 1\end{bmatrix} w_{k-1},$$

where

$$F(\omega) = \begin{bmatrix} 1 & \frac{\sin\omega T}{\omega} & 0 & -\frac{1-\cos\omega T}{\omega} \\ 0 & \cos\omega T & 0 & -\sin\omega T \\ 0 & \frac{1-\cos\omega T}{\omega} & 1 & \frac{\sin\omega T}{\omega} \\ 0 & \sin\omega T & 0 & \cos\omega T\end{bmatrix},\ G = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T\end{bmatrix},$$

$e_5 = [0,0,0,0,1]^T$, w_k is a 3-dimensional random vector following an i.i.d. white Gaussian process with zero mean and covariance $\text{Diag}([\sigma_w^2, \sigma_w^2, \sigma_\omega^2])$. In this example we use $T = 1\text{s}$, $\sigma_w = 50\text{m/s}^2$, and the standard deviation for the turn rate $\sigma_\omega = 0.35\text{rad/s}$.

Each new born target has initial state distributed according to a Gaussian with mean and covariance

$$\bar{x} = \begin{bmatrix} 1000\text{m} \\ 0\text{m/s} \\ 1500\text{m} \\ 0\text{m/s} \\ 0\text{rad/s}\end{bmatrix},\ \bar{Q} = \text{Diag}\left(\begin{bmatrix} 100\text{m}^2 \\ 500(\text{m/s})^2 \\ 100\text{m}^2 \\ 500(\text{m/s})^2 \\ (\pi/6)^2(\text{rad/s})^2\end{bmatrix}\right).$$

The number of target births follows a Poisson distribution with an average rate of 0.5 targets (per scan). The probability of target survival is $e_{k+1|k}(x_k) = 0.95$ for any x_k. For simplicity, we assume no spawning of targets.

The simulation settings for the sensor are as follows. We use $p_D(x_k) = 1$, i.e. the sensor does not miss any target generated measurements. The measurement errors for the targets [cf., Eq. (3)] are i.i.d. zero-mean Gaussian distributed. The variances of the range and bearing measurement errors are $(100\mathrm{m})^2$ and $(0.01\mathrm{rad/s})^2$, respectively. Clutter are uniformly distributed over the bearing region $[-\pi, \pi]$ and the range region $[0, 5000\mathrm{m}]$. The number of clutter points per scan is Poisson distributed with an average rate of 10.

Fig. 1 plots the true trajectories of the targets on top of the cluttered measurements. Fig. 2 shows the coordinate estimates of the particle-PHD filter. The particle parameters for the particle-PHD filter are $J_k = 4000$ and $L_k = 5000\lceil \hat{N}_{k|k} \rceil$, respectively. We see that the PHD filter estimates closely follow the true target tracks. To further examine the estimation accuracy of PHD filtering, Fig. 3 shows the PHD filter estimates with respect to time. Fig. 3(a) shows that the PHD filter does make errors on estimating the number of targets, but this only happens occasionally. Moreover, we observe from Figs. 3(b) and (c) that the multi-target coordinate tracking of the PHD filter is generally accurate, except for several occasions where the incorrect estimates of the number of targets lead to spurious target estimates. Similar results were found for the velocity and acceleration estimates of the PHD filter (they are not shown here due to space limitation).

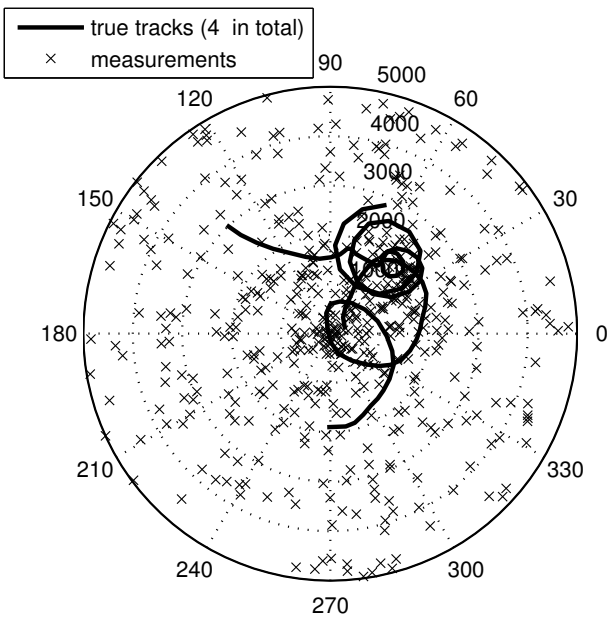

Fig. 1. True target trajectories and measured data.

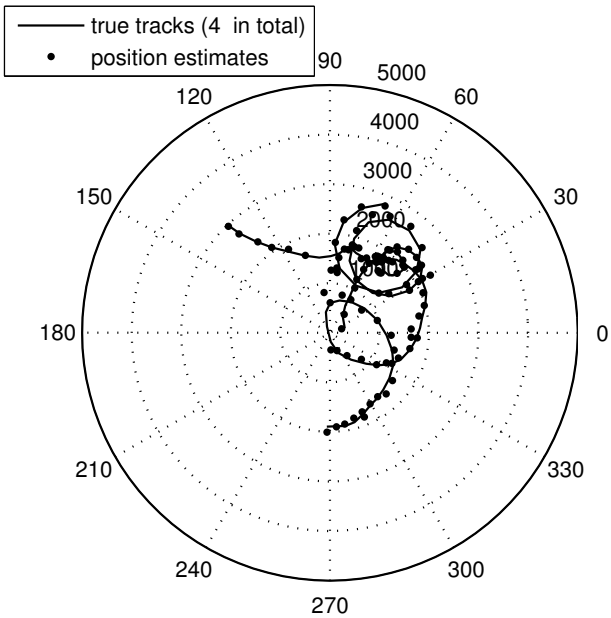

Fig. 2. PHD filter position estimates.

4.2. Multi-target tracking performance metric

In single target tracking, the root mean squares error is a standard measure for evaluating the performance of a filter. Unfortunately, this is not the case in multi-target tracking due to the mismatch between actual and estimated target numbers. Moreover, the association between true and estimated target states is not known. It has been proposed in [7] to use the Wasserstein distance as a multi-target miss-distance. Given the true and estimated sets of the target states, denoted by X_k and $\hat{X}_k$ respectively, the Wasserstein miss-distance is defined by

$$d_p(\hat{X}_k, X_k) = \min_C \sqrt[p]{\sum_{i=1}^{|\hat{X}_k|} \sum_{j=1}^{|X_k|} C_{i,j} \, \|\hat{x}_{i,k} - x_{j,k}\|^p} \quad (20)$$

where the minimum is taken over the set of all transportation matrices C, (a transportation matrix is one whose entries $C_{i,j}$ satisfy $C_{i,j} \geq 0, \sum_{j=1}^{|X_k|} C_{i,j} = 1/|\hat{X}_k|, \sum_{i=1}^{|\hat{X}_k|} C_{i,j} = 1/|X_k|$). Note that the Wasserstein miss-distance is not defined if either the estimate $\hat{X}_k$ or the ground truth X_k is an empty set. The optimization in (20) can be numerically done using linear programming.

Fig. 4 shows the Wasserstein miss-distance for the simulation example in the previous subsection, with $p = 2$. Notice that the large peaks in the Wasserstein miss-distance corresponds to the situations when there is an error with the number of target estimates; see Fig. 3(a). When there is no error with the number of target estimates, the Wasserstein miss-distance is roughly 100m, which coincides with the

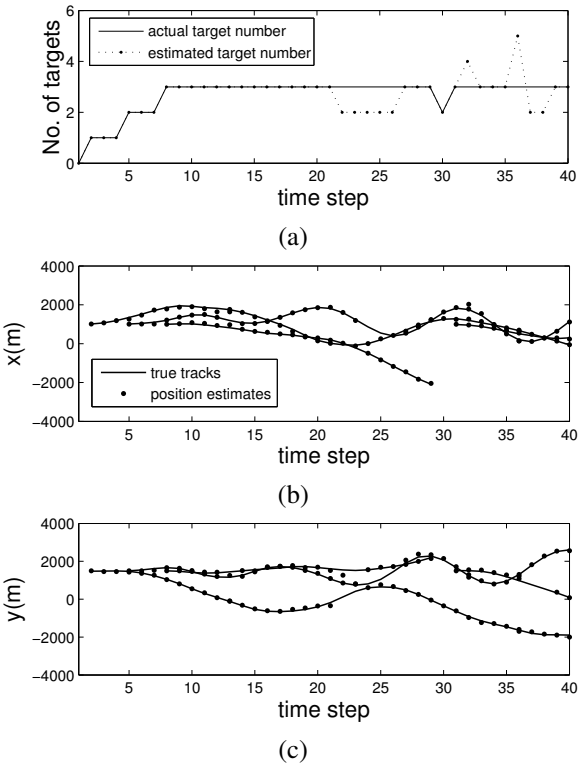

Fig. 3. PHD filter estimates with respect to time.

standard deviation of the range measurement noise. This implies that the Wasserstein miss-distance is a good indicator for mismatch between true and estimated multi-target states.

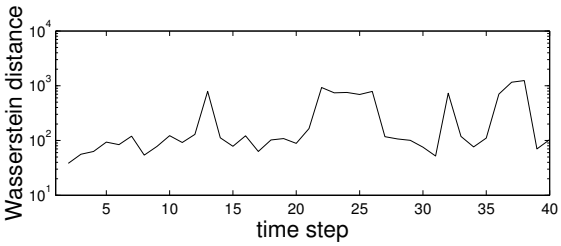

Fig. 4. Wasserstein miss-distance of the PHD filter.

5. CONCLUSION

In this paper, a set-valued Bayesian filtering framework for joint detection and tracking of multiple maneuvering targets with cluttered measurements has been presented. Our numerical results show that the PHD filter is capable of accurately estimating the motions of multiple maneuvering targets that appear and disappear at unknown time instants, even for mild clutter rates.

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