Particle Markov Chain Monte Carlo for Bayesian Multi-target Tracking

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Abstract— **We propose a new multi-target tracking (MTT) algorithm capable of tracking an unknown number of targets that move close and/or cross each other in a dense environment. The optimal Bayes MTT problem is formulated in the Random Finite Set framework and Particle Markov Chain Monte Carlo (PMCMC) is applied to compute the multi-target posterior distribution. The PMCMC technique is a combination of Markov chain Monte Carlo (MCMC) and sequential Monte Carlo methods to design an efficient high dimensional proposal distributions for MCMC algorithms. This technique allows our multi-target tracker to handle high track densities in a computationally feasible manner. Our simulations show that under scenarios with a large number of closely spaced tracks the estimated number of tracks and their trajectories are reliable.**

Keywords: Multi-target Tracking, Particle Markov chain Monte Carlo, Markov Chain Monte Carlo, Random Sets, Sequential Monte Carlo.

I. Introduction

The Multi-target tracking (MTT) problem is essentially that of estimating the presence and associated time trajectories of moving objects based on measurements from a variety of sensors. Over the past 40 years MTT has emerged as a fundamentally important technology with diverse applications ranging from radar tracking for aircraft, passive and active acoustic tracking for seacraft, video tracking of people for security applications, etc. [1], [2]

This paper investigates the problem where the targets being tracked may randomly appear and disappear from the field of view, they may be temporarily obscured by other objects, may merge and split, and may cross or travel very close to each other for extended periods of time. Sensor measurements also present a number of challenging characteristics, such as noise which introduces location errors and may cause missed detection of objects, false measurements which do not belong to a valid object of interest, ghosting, misidentification etc.

The majority of existing MTT algorithms such as Multiple Hypothesis Tracking (MHT) [3], Joint Probabilistic Data Association (JPDA) [4], Joint Integrated Probabilistic Data Association - JIPDA [5], and their variants etc have been found to perform effectively provided the density of targets and the number of false measurements are modest. However, these techniques are no longer adequate when the density of targets is high and the number of false measurements is large. In this paper we propose a Bayesian multi-target batch processing tracking algorithm based on Random Finite Set (RFS) modeling and Particle Markov Chain Monte Carlo (PMCMC) numerical approximation to overcome this problem.

Markov Chain Monte Carlo (MCMC) is a powerful computational tool for analysis of complex statistical problems. In the context of MTT, the MCMC was first used by Pasula et al [6], [7] to solve a multi-camera traffic surveillance problem involving hundreds of vehicles. Cong et al used MCMC to approximate the association probabilities in the JPDA filter [8]. The latest development in MCMC is PMCMC [9] which combines the strength of MCMC and Sequential Monte Carlo (SMC) to provide more versatile proposal distribution design.

The RFS approach to multi-target tracking is an emerging and promising alternative to the traditional association-based methods [10], [11]. The rationale for the RFS approach traces back to the fundamental notion of multi-target estimation error, see [12]. Moreover, in recent years this approach has attracted substantial interest due to theoretical and algorithmic advances such as the probability hypothesis density/cardinality probability hypothesis density (PHD/CPHD) filters [11], [13]–[16]. While these approximations are an improvement over traditional approaches in high density scenarios, performance can be improved with batch processing techniques, albeit at the expense of computation.

The main contribution of this paper is the development of a Bayesian multi-target batch processing tracking algorithm based on RFS modeling and PMCMC numerical approximation with the Gaussian Mixture Probability Hypothesis Density (GM-PHD) initialization, capable of tracking multiple targets in very high density situations.

The structure of this paper is as follows. Section II-A describes a model of a multi-target system. An RFS formulation of the multi-target tracking problem is presented in Section II-B. The Particle Marginal Metropolis-Hastings (PMMH) algorithm for MTT, is presented in Section III. Section IV gives the main result of this paper including simulations and discussion.

II. Problem formulation

This section presents a Bayesian formulation of the multi-target tracking problem. Subsection II-A describes the multi-target system model. Subsection II-B is devoted to the problem formulation in which the trajectories of targets (or tracks) are defined in Definition 1, a particular collection of tracks (track hypothesis) is defined in Definition 2 and finally a Bayesian recursion for construction of the posterior distribution of the tracks is presented.

We use the notation $u_{1:t} = (u_1, \ldots, u_t)$. Moreover, $|A|$ denotes the cardinality of the set A.

A. Random Finite Set Model

Let T be the number of measurement scans. Then $\mathcal{T} = \{1, ..., T\}$ is the set of time indices. In MTT problems, at time t, a multi-target state and a multi-target measurement are respectively represented as finite sets X_t and Z_t. If n_t targets are present at time t, the multi-target state $X_t = \{x_1, x_2, \ldots, x_{n_t}\} \subset \mathcal{X}$ where $\mathcal{X} \subseteq \mathbb{R}^{n_x}$ is the single-target state space. Similarly, if there are m_t observations at time t, the multi-target observation $Z_t = \{z_1, \ldots, z_{m_t}\} \subset \mathcal{Z}$ where $\mathcal{Z} \subseteq \mathbb{R}^{n_z}$ is the measurement space.

In order to distinguish multiple target trajectories we augment each single-target state with the target label. Thus our augmented single-target state space is

$$\widetilde{\mathcal{X}} = \mathcal{X} \times \mathbb{N}. \tag{1}$$

where $\mathbb{N} = \{1, 2, \ldots\}$.

Hereafter, if there is no ambiguity the state space and augmented state space are used interchangeably when referring to $\widetilde{\mathcal{X}}$. At time t, we denote the augmented multi-target state by $\widetilde{X}_t$. Note that $\widetilde{X}_t \in \mathcal{F}(\widetilde{\mathcal{X}})$ where $\mathcal{F}(A)$ denotes the collection of all finite subsets of the set A.

Each state $\tilde{x}_{t-1} \in \widetilde{X}_{t-1}$ is assumed to follow a Markov process in the following sense. The target either continues to exist at time t with probability $p_{S_t}(\tilde{x}_{t-1})$ and moves to the new state $\tilde{x}_t$ with probability density $\bar{f}_{t|t-1}(\tilde{x}_t|\tilde{x}_{t-1})$ or dies with probability $1 - p_{S_t}(\tilde{x}_{t-1})$ and takes on the value $\emptyset$. Thus, given an augmented single state $\tilde{x}_{t-1} \in \widetilde{X}_{t-1}$ at time $t-1$, its behavior at time t is modeled by the Bernoulli RFS

$$\mathrm{S}_{t|t-1}(\tilde{x}_{t-1})$$

that is either $\{\tilde{x}_t\}$ when the target survives or $\emptyset$ when the target dies. A new target at time t may result from either the spontaneous birth (i.e. independent of the surviving targets) or spawning from the target at time $t-1$. Note that labels for new targets cannot be chosen from existing targets or dead targets. The augmented multi-target state at time t is the union of the existing targets, the spawned targets and the spontaneous births

$$\widetilde{X}_t = \mathcal{S}_{t|t-1}(\widetilde{X}_{t-1}) \cup \mathcal{B}_{t|t-1}(\widetilde{X}_{t-1}) \cup \Gamma_t.$$

The three RFSs on the right hand side are assumed to be mutually independent conditional on $\widetilde{X}_{t-1}$ where $\mathcal{S}_{t|t-1}(\widetilde{X}_{t-1}) = \bigcup_{\tilde{x} \in \widetilde{X}_{t-1}} \mathrm{S}_{t|t-1}(\tilde{x})$ and $\mathcal{B}_{t|t-1}(\widetilde{X}_{t-1}) = \bigcup_{\tilde{x} \in \widetilde{X}_{t-1}} B_{t|t-1}(\tilde{x})$. The actual forms of $B_{t|t-1}(\tilde{x})$ and Γ_t are problem dependent. The augmented multi-target state transition can be alternatively stated in the form of an *augmented multi-target transition density* $f_{t|t-1}(\cdot|\cdot)$ given by

$$f_{t|t-1}(\widetilde{X}_t|\widetilde{X}_{t-1}) = (\pi_{\mathcal{S},t|t-1} * \pi_{\mathcal{B},t|t-1} * \pi_{\Gamma,t})(\widetilde{X}_t|\widetilde{X}_{t-1})$$

where

- $\pi_{\mathcal{S},t|t-1}(\cdot|\widetilde{X}_{t-1})$, $\pi_{\mathcal{B},t|t-1}(\cdot|\widetilde{X}_{t-1})$ and $\pi_{\Gamma,t}$ are the probability densities of the RFS of survival, spawning and spontaneous birth Γ_t; and
- $\beta * \gamma$ denotes the convolution between β and γ (see [13, chapter 13] for definition of this operator).

$f_{t|t-1}(\widetilde{X}_t|\widetilde{X}_{t-1})$ is initialized with the prior density μ_0 i.e. $f_{1|0}(\widetilde{X}_1|\widetilde{X}_0) = \mu_0(\widetilde{X}_1)$.

At time t, each augmented single-target state $\tilde{x}_t \in \widetilde{X}_t$, is either detected with probability $p_{D_t}(\tilde{x}_t)$ and generates an observation z_t with likelihood $\bar{g}_t(z_t|\tilde{x}_t)$, or missed with probability $1 - p_{D_t}(\tilde{x}_t)$. Thus, at time t, each augmented single-target state $\tilde{x}_t \in \widetilde{X}_t$ generates an RFS $D_t(\tilde{x}_t)$ that can take either the value $\{z_t\}$ when the target is observed by a sensor or $\emptyset$ when the target is not detected. Therefore target-generated measurements is

$$\mathfrak{D}_t(\widetilde{X}_t) = \bigcup_{\tilde{x} \in \widetilde{X}_t} D_t(\tilde{x}).$$

We assume that

(A.1) No two different targets share the same measurement at any time.

If more than two targets generates the same measurement, then this measurement will be arbitrarily associated with one of these targets and the other targets will be considered as not detected.

Apart from target-originated measurements, the sensor also receives a set of false/spurious measurements or clutter which is modeled by an RFS Λ_t. Thus, Z_t, the measurement at time t, is the union of target-generated measurements and clutter, hence

$$Z_t = \mathfrak{D}_t(\widetilde{X}_t) \cup \Lambda_t.$$

Given the probability density $\pi_{\mathfrak{D},t}(\cdot|\cdot)$ of target-generated measurement $\mathfrak{D}_t(\cdot)$ and the probability density $\pi_{\Lambda,t}(\cdot)$ of clutter Λ_t, the augmented multi-target likelihood is given by

$$g_t(Z_t|\widetilde{X}_t) = (\pi_{\mathfrak{D},t} * \pi_{\Lambda,t})(Z_t|\widetilde{X}_t). \tag{2}$$

For notational simplicity, $f(\widetilde{X}_t|\widetilde{X}_{t-1})$ and $g(Z_t|\widetilde{X}_t)$ are used in place of $f_{t|t-1}(\widetilde{X}_t|\widetilde{X}_{t-1})$, and $g_t(Z_t|\widetilde{X}_t)$ respectively if there is no ambiguity.

B. Track in RFS framework

Our objective is to estimate the tracks (paths of targets) over time. In terms of the states, a track is the collection of at least m^* single states on consecutive times with the same label where m^* is called a track gate. Mathematically, a track is defined as follows

Definition 1 (*Track*) *Given a track gate m^*, a track τ is an array of the form*

$$\tau = (k, t, x_0, \dots, x_m), \qquad m \geq m^* - 1 \tag{3}$$

where $k \in \mathbb{N}$ is the track label or identity, $t \in \mathcal{T}$ is the initial time of the track, $x_i \in \mathcal{X}$ is state of the track at time $t+i$ for $i = 0, \dots, m$.

For the track τ in (3), we denote the instances of track existence, the initial time of the track, the last existing time of the track and the track label respectively by

$$\begin{aligned}\mathfrak{T}(\tau) &= \{t, t+1, \dots, t+m\},\\ \mathfrak{T}_0(\tau) &= t,\\ \mathfrak{T}_f(\tau) &= t+m,\\ \mathcal{L}(\tau) &= k.\end{aligned}$$

For $t' \in \mathfrak{T}(\tau)$, we denote the state at time t' and the augmented state at time t' respectively by

$$\begin{aligned}\mathbf{x}_{t'}(\tau) &= x_{t'-t},\\ \tilde{\mathbf{x}}_{t'}(\tau) &= (x_{t'-t}, k).\end{aligned}$$

Similarly, for intuitive notation, given an augmented single-target state $\tilde{x} = (x, k)$ we also denote the label of $\tilde{x}$ by

$$\mathcal{L}(\tilde{x}) = k.$$

A collection of tracks in which no two tracks share the same state at any time is called a track Hypothesis.

Definition 2 (*Track hypothesis*) *A track hypothesis ω is a set of tracks such that no two tracks share the same label and no two tracks share the same state at any time i.e. for all $\tau, \tau' \in \omega$*

1) $\mathcal{L}(\tau) \neq \mathcal{L}(\tau')$ *and*
2) $\mathbf{x}_t(\tau) \neq \mathbf{x}_t(\tau')$ *for any* $t \in \mathfrak{T}(\tau) \cap \mathfrak{T}(\tau')$.

For a track hypothesis ω, we denote augmented multi-target state at time $t \in \mathcal{T}$ by

$$\widetilde{\mathbf{X}}_t(\omega) \quad = \quad \{\tilde{\mathbf{x}}_t(\tau) : \tau \in \omega\}.$$

Our objective is to find the track hypothesis ω which maximize the posterior distribution $p(\omega|Z_{1:T})$. Note that a track hypothesis ω can be equivalently specified by the array of augmented multi-target states $\widetilde{\mathbf{X}}_{1:T}(\omega) = (\widetilde{\mathbf{X}}_1(\omega), \dots, \widetilde{\mathbf{X}}_T(\omega))$. Hence,

$$p(\omega|Z_{1:T}) = p(\widetilde{X}_{1:T}|Z_{1:T}) = \frac{\prod_{t=1}^{T} f(\widetilde{X}_t|\widetilde{X}_{t-1}) g(Z_t|\widetilde{X}_t)}{p(Z_{1:T})} \tag{4}$$

where $\widetilde{X}_{1:T} = \widetilde{\mathbf{X}}_{1:T}(\omega) = (\widetilde{X}_1, \dots, \widetilde{X}_T)$ and $\widetilde{X}_t = \widetilde{\mathbf{X}}_t(\omega)$ for $t = 1, \dots, T$.

In the next section we propose a numerical approximation to the multi-target posterior distribution using the Particle Marginal - Metropolis Hastings (PMMH) sampler which is one of the PMCMC methods.

III. Algorithm

In this section we detail the main algorithm of the paper, namely PMMH Algorithm for MTT that combines PMMH Sampler [9] with proposal moves designed to deal with the varying dimensions of the multi-target state space.

Direct application of MCMC to the above form of the posterior distribution is intractable because computation of the likelihood function in (4) is intractable when the set of measurements and/or the number of target states at time t is large because all possible combination between target states and measurement must be considered. To overcome this problem, we employ a form of the multi-target likelihood given in [13].

$$g(Z_t|\widetilde{X}_t) \propto \sum_{\theta_t} g(z_1, \dots, z_{|Z_t|}|\widetilde{X}_t, \theta_t) w(\theta_t)$$

where θ_t is an auxiliary variable, defined as a mapping from $\mathcal{L}(\widetilde{X}_t)$ to $\{0, 1, \dots, |Z_t|\}$ with the property that $\theta_t(k) = \theta_t(k') > 0$ implies that $k = k'$;

$$g(z_{1:|Z_t|}|\widetilde{X}_t, \theta_t) = \prod_{i \notin \theta_t(\mathcal{L}(\widetilde{X}_t))} \frac{\kappa_t(z_i)}{\langle \kappa_t, 1 \rangle} \prod_{\tilde{x}' \in \widetilde{X}_t : \theta_t(\mathcal{L}(\tilde{x}'))=0} (1 - p_{D_t}(\tilde{x}')) \times \prod_{\tilde{x} \in \widetilde{X}_t : \theta_t(\mathcal{L}(\tilde{x}))>0} p_{D_t}(\tilde{x}) \bar{g}_t(z_{\theta_t(\mathcal{L}(\tilde{x}))}|\tilde{x}) \tag{5}$$

where κ_t denotes the clutter intensity at time t; $\langle u, v \rangle = \int u(x)v(x)dx$; and $g(z_{1:|Z_t|}|\widetilde{X}_t, \theta_t)$ in (5) is 1 if $Z_t = \emptyset$ (i.e. all targets are undetected if $\widetilde{X}_t \neq \emptyset$) or $\prod_{z \in Z_t} \frac{\kappa_t(z)}{\langle \kappa_t, 1 \rangle}$ if $\widetilde{X}_t = \emptyset$ (i.e. all measurements are clutter if $Z_t \neq \emptyset$); and

$$w(\theta_t) = e^{-\langle \kappa_t, 1 \rangle} \langle \kappa_t, 1 \rangle^{|\{1, \dots, |Z_t|\} - \{j : j \in \theta_t(\mathcal{L}(\widetilde{X}_t))\}|}.$$

where $w(\theta_t) = e^{-\langle \kappa_t, 1 \rangle} \langle \kappa_t, 1 \rangle^{|Z_t|}$ if $\widetilde{X}_t = \emptyset$. The auxiliary variable θ_t can be interpreted as the assignment of the target labels to the measurement indices. Undetected targets are assigned to 0. We extend the auxiliary variable θ_t to an augmented auxiliary variable $\tilde{\theta}_t$ by $\emptyset$ if $\widetilde{X}_t = \emptyset$ otherwise

$$\tilde{\theta}_t(k) = (\theta_t(k), k)$$

where $k \in \mathcal{L}(\widetilde{X}_t)$. We also denote $\tilde{\theta}_{1:T} = (\tilde{\theta}_1, \dots, \tilde{\theta}_T)$, $\widetilde{Z}_t = z_{1:|Z_t|}$ and $\widetilde{Z}_{1:t} = (\widetilde{Z}_1, \dots, \widetilde{Z}_t)$.

For notational simplicity Z, $\widetilde{Z}$, $\widetilde{X}$, $\tilde{\theta}$ are used in place of $Z_{1:T}$, $\widetilde{Z}_{1:T}$, $\widetilde{X}_{1:T}$, $\tilde{\theta}_{1:T}$ respectively if there is no ambiguity. The multi-target posterior distribution now takes the form

$$p(\widetilde{X}|Z) \propto \sum_{\tilde{\theta}} p(\widetilde{X}|\widetilde{Z}, \tilde{\theta}) p(\tilde{\theta}|\widetilde{Z}) \tag{6}$$

where

$$p(\tilde{\theta}|\widetilde{Z}) \propto p(\widetilde{Z}|\tilde{\theta}) w(\tilde{\theta})$$

with $w(\tilde{\theta}) = \prod_{t=1}^{T} w(\tilde{\theta}_t)$, $w(\tilde{\theta}_t) = w(\theta_t)$; and

$$p(\widetilde{X}|\widetilde{Z}, \tilde{\theta}) = \frac{p(\widetilde{X}, \widetilde{Z}|\tilde{\theta})}{p(\widetilde{Z}|\tilde{\theta})} = \frac{\prod_{t=1}^{T} f(\widetilde{X}_t|\widetilde{X}_{t-1}) g(\widetilde{Z}_t|\widetilde{X}_t, \tilde{\theta}_t)}{p(\widetilde{Z}|\tilde{\theta})}.$$

with $g(\widetilde{Z}_t|\widetilde{X}_t, \tilde{\theta}_t) = g(\widetilde{Z}_t|\widetilde{X}_t, \theta_t)$. The variable $\tilde{\theta}$ is in essence a nuisance variable to be marginalized out. Our aim is to find $\tilde{\theta}$ and $\widetilde{X}$ that maximizes $p(\widetilde{X}, \tilde{\theta}|\widetilde{Z})$. The right hand side of (6) suggests that for each MC iteration we sample $\tilde{\theta}$ first and then sample $\widetilde{X}$ conditioned on $\tilde{\theta}$ and $\widetilde{Z}$. This approach is called Marginal Metropolis-Hastings (MMH) sampling. The Particle Marginal - Metropolis Hastings (PMMH) sampler [9] uses SMC approximation as a proposal distribution for the Metropolis-Hastings (MH) sampler.

A. PMMH for Multi-target Tracking

Given $\tilde{\theta}$ and $\widetilde{Z}$, the SMC algorithm propagates N particles $\{\widetilde{X}_{1:t}^n, W_t(\widetilde{X}_{1:t}^n)\}_{n=1}^N$ $t = 1, \ldots, T$ as follows.

At time $t = 1$: the importance sampling (IS) is used to approximate $p(\widetilde{X}_1|\widetilde{Z}_1, \tilde{\theta}_1)$ by using an importance density $q(\widetilde{X}_1|\widetilde{Z}_t, \tilde{\theta}_1)$ to sample N particles $\{\widetilde{X}_1^n, W_1^n\}_{n=1}^N$. Then resampling step is used to sample N times from the IS approximation $\widehat{p}(\widetilde{X}_1|\widetilde{Z}_t, \tilde{\theta}_1)$ of $p(\widetilde{X}_1|\widetilde{Z}_t, \tilde{\theta}_1)$. N samples $\{\widetilde{X}_1^n\}_{n=1}^N$ which are obtained from resampling step are approximately distributed according to $p(\widetilde{X}_1|\widetilde{Z}_t, \tilde{\theta}_1)$.

At time $t = 2, \ldots, T$: the posterior distribution

$$p(\widetilde{X}_{1:t}|\widetilde{Z}_{1:t}, \tilde{\theta}_{1:t}) \propto p(\widetilde{X}_{1:t-1}|\widetilde{Z}_{1:t-1}, \tilde{\theta}_{1:t-1}) \times f(\widetilde{X}_t|\widetilde{X}_{t-1}) g(\widetilde{Z}_t|\widetilde{X}_t, \tilde{\theta}_t)$$

suggests that the samples at previous time $t-1$ which approximate the posterior distribution $p(\widetilde{X}_{1:t-1}|\widetilde{Z}_{1:t-1}, \tilde{\theta}_{1:t-1})$ can be used at time step t by extending each of such these particles through the IS distribution $q(\widetilde{X}_t|\widetilde{Z}_t, \widetilde{X}_{t-1}, \tilde{\theta}_t)$ to produce samples approximately distributed according to $p(\widetilde{X}_{1:t-1}|\widetilde{Z}_{1:t-1}, \tilde{\theta}_{1:t-1}) q(\widetilde{X}_t|\widetilde{X}_{t-1}, \tilde{\theta}_t)$. Then the resampling step produces samples $\{\widetilde{X}_{1:t}^n\}_{n=1}^N$ approximately distributed according to $p(\widetilde{X}_{1:t}|\widetilde{Z}_{1:t}, \tilde{\theta}_{1:t})$.

The pseudocode of the SMC algorithm is given in Algorithm 1. $\mathbf{W}_t = (W_t^1, \ldots, W_t^N)$ defines a probability distribution on $\{1, \ldots, N\}$ denoted by $\mathfrak{F}(\cdot|\mathbf{W}_t)$.

In Algorithm 1, for $n = 1, \ldots, N$ the variable A_{t-1}^n is the index of the 'parent' at time $t-1$ of particle $\widetilde{X}_{1:t}^n$ for $t = 2, \ldots, T$. The variables $B_{1:T}^n$ is introduced as the ancestral lineage of the particle $\widetilde{X}_{1:t}^n$ such that $B_T^n = n$ and $B_t^n = A_t^{B_{t+1}^n}$ for $t = T-1, \ldots, 1$. Therefore, particle $\widetilde{X}_{1:T}^n = (\widetilde{X}_1^{B_1^n}, \ldots, \widetilde{X}_T^{B_T^n})$ for $n = 1, \ldots, N$.

The SMC algorithm provides us an approximation of the posterior distribution $p(\widetilde{X}|\widetilde{Z}, \tilde{\theta})$ as follows

$$\widehat{p}(\widetilde{X}|\widetilde{Z}, \tilde{\theta}) = \sum_{n=1}^N W_T^n \delta(\widetilde{X} - \widetilde{X}_{1:T}^n) \quad (9)$$

where $\delta(\cdot)$ is a dirac delta function. In addition, the estimate of the marginal likelihood $p(\widetilde{Z}|\tilde{\theta})$ is

$$\widehat{p}(\widetilde{Z}|\tilde{\theta}) = \prod_{t=1}^T \widehat{p}(\widetilde{Z}_t|\widetilde{Z}_{1:t-1}, \tilde{\theta}_{1:t}) \quad (10)$$

where $\widehat{p}(\widetilde{Z}_1|\widetilde{Z}_0, \tilde{\theta}_1) = \widehat{p}(\widetilde{Z}_1|\tilde{\theta}_1)$ and

$$\widehat{p}(\widetilde{Z}_t|\widetilde{Z}_{1:t-1}, \tilde{\theta}_{1:t}) = \frac{1}{N} \sum_{n=1}^N w_t(\widetilde{X}_{1:t}^n) \quad (11)$$

Algorithm 1 : SMC Algorithm

Input: Given $\widetilde{Z}$, $\tilde{\theta}$, p_{S_t}, p_{D_t}, κ_t, the birth intensity γ_t, for $t = 1, \ldots, T$ and sample number N.

Output: $\widetilde{X}_{1:T}^n, W_T^n$, and $w_t^n(\widetilde{X}_{1:t}^n)$ for $n = 1, \ldots, N$.

At time $t = 1$:

- sample $\widetilde{X}_1^n \sim q(\cdot|\widetilde{Z}_1, \tilde{\theta}_1)$. Then compute

$$w_1(\widetilde{X}_1^n) = \frac{p(\widetilde{X}_1^n, \widetilde{Z}_1|\tilde{\theta}_1)}{q(\widetilde{X}_1^n|\widetilde{Z}_1, \tilde{\theta}_1)} = \frac{\mu_0(\widetilde{X}_1^n) g(\widetilde{Z}_1|\widetilde{X}_1^n, \tilde{\theta}_1)}{q(\widetilde{X}_1^n|\widetilde{Z}_1, \tilde{\theta}_1)} \quad (7)$$

and normalize $W_1^n = w_1(\widetilde{X}_1^n) / \sum_{m=1}^N w_1(\widetilde{X}_1^m)$.

At $t = 2, \ldots, T$:

- sample $A_{t-1}^n \sim \mathfrak{F}(\cdot|\mathbf{W}_{t-1})$, then $\widetilde{X}_t^n \sim q(\cdot|\widetilde{X}_{t-1}^{A_{t-1}^n}, \widetilde{Z}_t, \tilde{\theta}_t)$ and set $\widetilde{X}_{1:t}^n = (\widetilde{X}_{1:t-1}^{A_{t-1}^n}, \widetilde{X}_t^n)$. Then compute

$$\begin{aligned} w_t(\widetilde{X}_{1:t}^n) &= \frac{p(\widetilde{X}_{1:t}^n, \widetilde{Z}_{1:t}|\tilde{\theta}_{1:t})}{p(\widetilde{X}_{1:t-1}^{A_{t-1}^n}, \widetilde{Z}_{1:t-1}|\tilde{\theta}_{1:t-1}) q(\widetilde{X}_t^n|\widetilde{X}_{t-1}^{A_{t-1}^n}, \widetilde{Z}_t, \tilde{\theta}_t)} \\ &= \frac{f(\widetilde{X}_t^n|\widetilde{X}_{t-1}^{A_{t-1}^n}) g(\widetilde{Z}_t|\tilde{\theta}_t, \widetilde{X}_t^n)}{q(\widetilde{X}_t^n|\widetilde{X}_{t-1}^{A_{t-1}^n}, \widetilde{Z}_t, \tilde{\theta}_t)} \end{aligned} \quad (8)$$

and normalize $W_t^n = w_t(\widetilde{X}_{1:t}^n) / \sum_{m=1}^N w_t(\widetilde{X}_{1:t}^m)$.

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is an estimate at time t of

$$p(\widetilde{Z}_t|\widetilde{Z}_{1:t-1}, \tilde{\theta}_{1:t}) = \int w_t(\widetilde{X}_{1:t}) q(\widetilde{X}_t|\widetilde{Z}_t, \widetilde{X}_{t-1}, \tilde{\theta}_t) \times p(\widetilde{X}_{1:t-1}|\widetilde{Z}_{1:t-1}, \tilde{\theta}_{1:t-1}) d\widetilde{X}_{1:t}. \quad (12)$$

In our problem, apart from estimating $\widetilde{X}$, we also need to estimate the unknown $\tilde{\theta}$ so sampling both $\widetilde{X}$ and the unknown $\tilde{\theta}$ from the posterior distribution $p(\tilde{\theta}, \widetilde{X}|\widetilde{Z})$ are required where

$$p(\tilde{\theta}, \widetilde{X}|\widetilde{Z}) \propto p(\widetilde{X}|\widetilde{Z}, \tilde{\theta}) p(\widetilde{Z}|\tilde{\theta}) w(\tilde{\theta}).$$

The MH algorithm is employed with a proposal distribution of the following form

$$\mathfrak{q}(\widetilde{X}^*, \tilde{\theta}^*|\widetilde{X}, \tilde{\theta}, \widetilde{Z}) = \mathbf{q}(\tilde{\theta}^*|\tilde{\theta}, \widetilde{Z}) p(\widetilde{X}^*|\widetilde{Z}, \tilde{\theta}^*).$$

Then MH acceptance rate in MMH algorithm is

$$\frac{p(\widetilde{X}^*, \tilde{\theta}^*|\widetilde{Z}) \mathfrak{q}(\widetilde{X}, \tilde{\theta}|\widetilde{X}^*, \tilde{\theta}^*, \widetilde{Z})}{p(\widetilde{X}, \tilde{\theta}|\widetilde{Z}) \mathfrak{q}(\widetilde{X}^*, \tilde{\theta}^*|\widetilde{X}, \tilde{\theta}, \widetilde{Z})} = \frac{p(\widetilde{Z}|\tilde{\theta}^*) w(\tilde{\theta}^*) \mathbf{q}(\tilde{\theta}|\tilde{\theta}^*, \widetilde{Z})}{p(\widetilde{Z}|\tilde{\theta}) w(\tilde{\theta}) \mathbf{q}(\tilde{\theta}^*|\tilde{\theta}, \widetilde{Z})}. \quad (13)$$

By using $\widehat{p}(\widetilde{X}|\widetilde{Z}, \tilde{\theta})$ and $\widehat{p}(\widetilde{Z}|\tilde{\theta})$ in place of $p(\widetilde{X}|\widetilde{Z}, \tilde{\theta})$ and $p(\widetilde{Z}|\tilde{\theta})$ respectively in MH update, the PMMH sampler is given in algorithm 2 for $l = 1, \ldots, L$.

B. Proposal design

In this section, we will show how to build the proposal distribution $\mathbf{q}(\cdot|\tilde{\theta}, \widetilde{Z})$ by constructing an MC on the space of $\tilde{\theta}$. Instead we construct an MC on an equivalent space. We define $\tilde{\theta}_\tau$ as a track auxiliary variable as follows

$$\tilde{\theta}_\tau = (k, t, j_0, \ldots, j_m) \quad (14)$$

Algorithm 2 : PMMH algorithm

Input: Given $\widetilde{Z}$, p_{S_t}, p_{D_t}, κ_t, the birth intensity γ_t for $t = 1, \ldots, T$ and sample number L.
Output: $S_X(l), S_{\tilde{\theta}}(l)$, and $\boldsymbol{\gamma_\theta}(l)$ for $l = 1, \ldots, L$.
At iteration $l = 1$:

- Set $\tilde{\theta}$ arbitrarily. Denote $S_{\tilde{\theta}}(l) = \tilde{\theta}$, then
- run an SMC algorithm targeting $p(\cdot|\widetilde{Z}, \tilde{\theta})$, sample $\widetilde{X} \sim \widehat{p}(\cdot|\widetilde{Z}, \tilde{\theta})$ and calculate $\widehat{p}(\widetilde{Z}|\tilde{\theta})$. Assign $S_X(l) = \widetilde{X}$ and $\boldsymbol{\gamma_\theta}(l) = \widehat{p}(\widetilde{Z}|\tilde{\theta})$.

At iteration $l > 1$:

- Propose $\tilde{\theta}^* \sim \mathbf{q}(\cdot|S_{\tilde{\theta}}(l-1), \widetilde{Z})$,
- run an SMC algorithm targeting $p(\cdot|\widetilde{Z}, \tilde{\theta}^*)$, sample $\widetilde{X}^* \sim \widehat{p}(\cdot|\widetilde{Z}, \tilde{\theta}^*)$ and calculate $\widehat{p}(\widetilde{Z}|\tilde{\theta}^*)$.
- calculate an acceptance rate
$$\alpha = \min\left\{1, \frac{\widehat{p}(\widetilde{Z}|\tilde{\theta}^*) w(\tilde{\theta}^*) \mathbf{q}(S_{\tilde{\theta}}(l-1)|\tilde{\theta}^*, \widetilde{Z})}{\boldsymbol{\gamma_\theta}(l-1) w(S_{\tilde{\theta}}(l-1)) \mathbf{q}(\tilde{\theta}^*|S_{\tilde{\theta}}(l-1), \widetilde{Z})}\right\}$$
- if $\alpha \geq u$ where $u \sim Unif[0,1]$, set $S_X(l) = \widetilde{X}^*$, $\boldsymbol{\gamma_\theta}(l) = \widehat{p}(\widetilde{Z}|\tilde{\theta}^*)$ and $S_{\tilde{\theta}}(l) = \tilde{\theta}^*$. Otherwise $S_X(l) = S_X(l-1), S_{\tilde{\theta}}(l) = S_{\tilde{\theta}}(l-1)$, $\boldsymbol{\gamma_\theta}(l) = \boldsymbol{\gamma_\theta}(l-1)$.

where $k = \mathcal{L}(\tau)$, $t = \mathfrak{T}_0(\tau)$ and $\tilde{\theta}_{t+i}(k) = (j_i, k)$ for $i = 0, \ldots, m$. Hence, the track auxiliary variable $\tilde{\theta}_\tau$ contains information about the measurement association with a track and has the same properties as track τ such as the same label i.e. $\mathcal{L}(\tilde{\theta}_\tau) = \mathcal{L}(\tau)$, the same existence duration time i.e. $\mathfrak{T}(\tilde{\theta}_\tau) = \mathfrak{T}(\tau)$, the initial time of appearance $\mathfrak{T}_0(\tilde{\theta}_\tau) = \mathfrak{T}_0(\tau)$ and the last time of existence $\mathfrak{T}_f(\tilde{\theta}_\tau) = \mathfrak{T}_f(\tau)$. For the track auxiliary variable $\tilde{\theta}_\tau$ in (14), we denote the measurement index of $\tilde{\theta}_\tau$ at time t' by

$$\mathfrak{I}_{t'}(\tilde{\theta}_\tau) = j_{t'-t}.$$

Hence the target (labeled) $\mathcal{L}(\tau)$ is undetected if $\mathfrak{I}_{t'}(\tilde{\theta}_\tau) = 0$ or generates the measurement $z_{\mathfrak{I}_{t'}(\tilde{\theta}_\tau)}$ if $\mathfrak{I}_{t'}(\tilde{\theta}_\tau) > 0$. We also denote

$$\tilde{\theta}_\omega = \left\{\tilde{\theta}_\tau : \tau \in \omega\right\}.$$

Then $\tilde{\theta}_\omega$ is called a track hypothesis auxiliary variable. $\tilde{\theta}_\omega$ and $\tilde{\theta}$ are equivalent representations of the combination between tracks and their measurements. Given $\tilde{\theta}_\omega$, for $t = 1, \ldots, T$, $\tilde{\theta}_t$ is defined by $\emptyset$ if $t \notin \bigcup_{\tilde{\theta}_\tau \in \tilde{\theta}_\omega} \mathfrak{T}(\tilde{\theta}_\tau)$ otherwise

$$\tilde{\theta}_t(\mathcal{L}(\tilde{\theta}_\tau)) = (\mathfrak{I}_t(\tilde{\theta}_\tau), \mathcal{L}(\tilde{\theta}_\tau)), \qquad \tilde{\theta}_\tau \in \tilde{\theta}_\omega. \tag{15}$$

Thus constructing an MC on the space of $\tilde{\theta}$ is equivalent to constructing an MC on the space of $\tilde{\theta}_\omega$. At time t we denote the clutter of track hypothesis ω by $\boldsymbol{\Lambda}_t(\omega) = \left\{z_j \in Z_t : j \notin \bigcup_{\tau \in \omega}\{\mathfrak{I}_t(\tilde{\theta}_\tau)\}\right\}$ and the probability going from $\tilde{\theta}_\omega$ to $\tilde{\theta}_{\omega^*}$ given $\widetilde{Z}$ by $\mathrm{q}(\tilde{\theta}_{\omega^*}|\widetilde{Z}, \tilde{\theta}_\omega)$, then $\mathbf{q}(\tilde{\theta}^*|\widetilde{Z}, \tilde{\theta}) = \mathrm{q}(\tilde{\theta}_{\omega^*}|\widetilde{Z}, \tilde{\theta}_\omega)$.

The proposal distribution $\mathrm{q}(\tilde{\theta}_{\omega^*}|\widetilde{Z}, \tilde{\theta}_\omega)$ is constructed using fourteen proposal moves (illustrated in figure 1) to generate an ergodic MC on the space of $\tilde{\theta}_\omega$ with $\bar{d}$ as the maximum number of consecutive missed detection of any targets. These

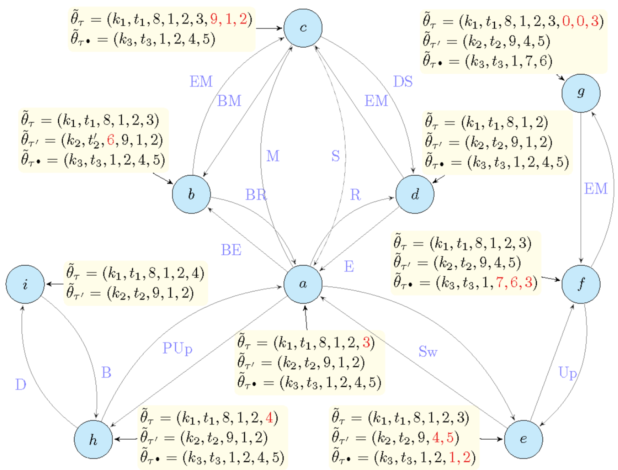

Fig. 1: Fourteen moves of the MC on the space of $\tilde{\theta}_\omega$ with track gate $m^* = 3$ and $\bar{d} = 2$ where $t_3 = t_1 + 3$, $t_2 = t_3 + 1$ and $t'_2 = t_2 - 1$. Each move proposes a new track hypothesis auxiliary variable $\tilde{\theta}_{\omega^*}$ that modifies the current track hypothesis auxiliary variable $\tilde{\theta}_\omega$. The Birth move $(i \to h)$ adds $\tilde{\theta}_{\tau\bullet}$ which is constructed from the set of clutter $\bigcup_{t \in \mathcal{T}} \boldsymbol{\Lambda}_t(\omega)$ to node (i) while the death move $(h \to i)$ removes $\tilde{\theta}_{\tau\bullet}$ at node (h). The split move $(c \to a)$ splits $\tilde{\theta}_\tau$ at node (c) while the Merge move $(a \to c)$ combines $\tilde{\theta}_\tau$ and $\tilde{\theta}_{\tau'}$ at node (a). The extension move $(d \to a)$ adds measurement index 3 after the last measurement index of $\tilde{\theta}_\tau$ at node (d) while the Reduction move $(a \to d)$ removes the last measurement index 3 from $\tilde{\theta}_\tau$ at node (a). Similarly, the Backward Extension move $(a \to b)$ adds measurement index 6 before the first measurement index of $\tilde{\theta}_{\tau'}$ at node (a) while the Backward Reduction $(b \to a)$ move removes the first measurement index 6 from $\tilde{\theta}_{\tau'}$ at node (b). The Switch move $(a \leftrightarrow e)$ exchanges measurement indices between $\tilde{\theta}_{\tau'}$ and $\tilde{\theta}_{\tau\bullet}$. The Extension Merge move $(b \to c)$ merges $\tilde{\theta}_\tau$ and $\tilde{\theta}_{\tau'}$ at node (b) but removes the first measurement index at $\tilde{\theta}_{\tau'}$ while the Birth Merge move $(c \to b)$ adds $\tilde{\theta}_{\tau'}$ at node (b) starting at measurement index 6 then merging to $\tilde{\theta}_\tau$ at node (c) starting from measurement index 9. The Extension Merge move $(d \to c)$ applies to $\tilde{\theta}_\tau$ and $\tilde{\theta}_{\tau'}$ at node (d) while Delete Split move $(c \to d)$ applies to $\tilde{\theta}_\tau$ at node (c). The Extension Merge move $(f \leftrightarrow g)$ applies to $\tilde{\theta}_\tau$ and $\tilde{\theta}_{\tau\bullet}$. The Update move $(e \leftrightarrow f)$ applies $\tilde{\theta}_{\tau\bullet}$ while the Point Update move $(a \leftrightarrow h)$ applies to $\tilde{\theta}_\tau$.

fourteen proposal moves are classified into ten groups

- Group I: Birth (B)/Death (D)
- Group II: Split (S)/Merge (M)
- Group III: Extension (E)/Reduction (R)
- Group IV: Backward Extension (BE)/Backward Reduction (BR),
- Group V: Switch (Sw),
- Group VI: Extension Merge (EM)/Birth Merge (BM),
- Group VII: Extension Merge (EM)/Delete Split (DS),
- Group VIII: Extension Merge (EM),

- Group IX: Update (Up),
- Group X: Point Update (PUp).

where the moves in groups I, II, III, V, and IX are from [17]. The moves from groups IV and X; and three moves from groups VI, VII and VIII are derived to speed up the convergence of the MC on the space of $\tilde{\theta}_\omega$. If a group consists of two moves, one of the moves is a reverse move of the other. If a group includes only one move, the move and its reverse move are the same. Denote $K_\omega = |\tilde{\theta}_\omega|$ as the number of tracks in the track hypothesis ω. The moves proposed on the space of $\tilde{\theta}_\omega$ are briefly explained as follows.

- If $K_\omega = 0$, only a birth move is proposed.
- If $K_\omega \neq 0$ and $\mathfrak{T}_f(\tilde{\theta}_\tau) = T$ (i.e. the last existing time of track τ is the last scan T), neither Extension move nor Extension Merge move of $\tilde{\theta}_\tau$ is proposed. When $|\mathfrak{T}(\tilde{\theta}_\tau)| = m^*$ (i.e. duration time of existence of track τ is not large than m^*), the Reduction or Backward Reduction move cannot occur for $\tilde{\theta}_\tau$. When $|\mathfrak{T}(\tilde{\theta}_\tau)| \leq 2m^*$, the Split or Split Delete moves cannot occur for $\tilde{\theta}_\tau$.
- If $K_\omega \neq 0$ and $\mathfrak{T}_0(\tilde{\theta}_\tau) = 1$ (i.e. the first appearance time of track τ is the first scan), no Backward Extension move of $\tilde{\theta}_\tau$ is proposed.
- If $K_\omega = 1$, neither Merge, Extension Merge nor Switch move occurs.
- In other situations, the moves are uniformly distributed.

Conditional on $\tilde{\theta}_\omega$, a track hypothesis auxiliary variable $\tilde{\theta}_{\omega^*}$ is chosen subject to the condition above, then $\tilde{\theta}^*$ is found by (15). By the construction of the proposal moves mentioned above, the acceptance rate in (13) can be written as

$$\frac{p(\widetilde{Z}|\tilde{\theta}^*)w(\tilde{\theta}^*)\mathbf{q}(\tilde{\theta}|\tilde{\theta}^*,\widetilde{Z})}{p(\widetilde{Z}|\tilde{\theta})w(\tilde{\theta})\mathbf{q}(\tilde{\theta}^*|\tilde{\theta},\widetilde{Z})} = \frac{p(\widetilde{Z}|\tilde{\theta}^*)w(\tilde{\theta}^*)}{p(\widetilde{Z}|\tilde{\theta})w(\tilde{\theta})}. \quad (16)$$

and $\tilde{\theta}_{\omega^*}$ specifies some track hypothesis ω^* and hence $\tilde{\theta}^*$ is the sequence of augmented auxiliary variables of $\widetilde{X}_{1:T}(\omega^*)$. Hence whenever $\widetilde{X} \sim q(\cdot|\widetilde{Z},\tilde{\theta}^*)$, there exists a track hypothesis ω^* such that $\widetilde{X} = \widetilde{\mathbf{X}}_{1:T}(\omega^*)$.

Initializing $\tilde{\theta}$ arbitrarily in Algorithm 2 makes the computation expensive. This can be alleviated by using estimate from GM-PHD tracker. Using a good estimate from GM-PHD tracker reduce the computation cost at least 20 times. Starting an MC with $\tilde{\theta}$ obtained from an estimate $\widetilde{X}^G$ from the GM-PHD tracker requires us to keep this estimate to reduce the computation cost. The SMC modified to suit this situation is called the conditional SMC [9] which only need to sample $N-1$ particles from $q(\cdot|\widetilde{Z},\tilde{\theta})$.

The pseudocode of SMC Algorithm 3 provides us with the parameters $B^n_{1:T}$ as the ancestral lineage of the particle $\widetilde{X}^n_{1:T}$. The conditional SMC algorithm conditional on $\widetilde{X}^k_{1:T} = (\widetilde{X}_1^{B_1^k},\ldots,\widetilde{X}^{B_T^k})$ is described in algorithm 3 to sample $N-1$ particles as follows for $n = 1,\ldots,N$.

Based on the PMMH Algorithm 2, the algorithm of PMMH for MTT is summarized in Algorithm 4

IV. Simulation and Performance

In this section, we demonstrate the multi-target PMMH algorithm with a simulated sample and evaluate its performance using the Optimal Sub-pattern Assignment distance (OSPA) [18]. The surveillance area is the square region $\mathcal{R} = [-1000m, 1000m] \times [-1000m, 1000m]$. We use the surveillance duration of $T = 50$ scans with sampling interval $T_s = 1$ second. We denote x_t^{Tr} is the transpose of x. The state vector is $x_t = [\xi_t, \zeta_t, \dot{\xi}_t, \dot{\zeta}_t]^{Tr}$ where (ξ_t, ζ_t) denotes the target position on 2D Cartesian plane and $(\dot{\xi}_t, \dot{\zeta}_t)$ is its velocity $t = 1,\ldots,T$. Linear state and measurement models are used

$$x_t = Ax_{t-1} + v_{t-1}, z_t = Cx_t + w_t \quad (17)$$

where

$$A = \begin{bmatrix} 1 & 0 & T_s & 0 \\ 0 & 1 & 0 & T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^{Tr},$$

Algorithm 3 : Conditional SMC Algorithm

At time $t = 1$:

- if $n \neq B_1^k$, sample $\widetilde{X}_1^n \sim q(\cdot|\widetilde{Z}_1,\tilde{\theta}_1)$ and compute $w_1(\widetilde{X}_1^n)$ by using (7) and normalize $W_1^n \propto w_1(\widetilde{X}_1^n)$.

At $t = 2,\ldots,T$:

- if $n \neq B_t^k$, sample $A^n_{t-1} \sim \mathfrak{F}(\cdot|\mathbf{W}_{t-1})$,
- then sample $\widetilde{X}_t^n \sim q(\cdot|\widetilde{X}_{t-1}^{A^n_{t-1}},\widetilde{Z}_t,\tilde{\theta}_t)$, set $\widetilde{X}^n_{1:t} = (\widetilde{X}^{A^n_{t-1}}_{1:t-1},\widetilde{X}^n_t)$ and
- compute $w_t(\widetilde{X}^n_{1:t})$ by using (8) and normalize $W_t^n \propto w_t(\widetilde{X}^n_{1:t})$.

Algorithm 4 :PMMH Algorithm for MTT

Input: Given $\widetilde{Z}$, p_{S_t}, p_{D_t}, κ_t, the birth intensity γ_t for $t = 1,\ldots,T$ and sample number L.
Output: $S_X(l), S_{\tilde{\theta}}(l)$, and $\boldsymbol{\gamma_\theta}(l)$ for $l = 1,\ldots,L$.

At iteration $l = 1$

- Run GM-PHD tracker to obtain $\widetilde{X}^G$, then obtains $\tilde{\theta}$ from $\widetilde{X}^G$ and denote $B_{1:T} = (1,\ldots,1)$.
- Run a conditional SMC algorithm targeting $p(\widetilde{X}|\widetilde{Z},\tilde{\theta})$ conditional on $\widetilde{X}^G$ and $B_{1:T}$. Then sample $\widetilde{X}^* \sim \widehat{p}(\cdot|\widetilde{Z},\tilde{\theta})$ and calculate $\boldsymbol{\gamma_\theta}(l) = \widehat{p}(\widetilde{Z}|\tilde{\theta})$. Then denote $S_X(l) = \widetilde{X}^*$, $S_{\tilde{\theta}}(l) = \tilde{\theta}$.

At iteration $l > 1$

- Propose $\tilde{\theta}^* \sim \mathbf{q}(\cdot|S_{\tilde{\theta}}(l-1),\widetilde{Z})$ described above in the proposal move.
- Run an SMC algorithm targeting $p(\widetilde{X}|\widetilde{Z},\tilde{\theta}^*)$. Then sample $\widetilde{X}^* \sim \widehat{p}(\cdot|\widetilde{Z},\tilde{\theta}^*)$; calculate $\widehat{p}(\widetilde{Z}|\tilde{\theta}^*)$
- By (16), the acceptance rate
$$\alpha = \min\left\{1, \frac{\widehat{p}(\widetilde{Z}|\tilde{\theta}^*)w(\tilde{\theta}^*)}{\boldsymbol{\gamma_\theta}(l-1)w(S_{\tilde{\theta}}(l-1))}\right\}$$
- if $\alpha \geq u$ where $u \sim Unif[0,1]$, set $S_X(l) = \widetilde{X}^*$, $\boldsymbol{\gamma_\theta}(l) = \widehat{p}(\widetilde{Z}|\tilde{\theta}^*)$ and $S_{\tilde{\theta}}(l) = \tilde{\theta}^*$. Otherwise $S_X(l) = S_X(l-1), S_{\tilde{\theta}}(l) = S_{\tilde{\theta}}(l-1)$, $\boldsymbol{\gamma_\theta}(l)$ =$\boldsymbol{\gamma_\theta}(l-1)$.

$$Q = \sigma_v^2 \begin{bmatrix} \frac{T_s^2}{4} I_2 & \frac{T_s}{2} I_2 \\ \frac{T_s}{2} I_2 & I_2 \end{bmatrix}, \; R = \sigma_w^2 I_2$$

and where v_t and w_t are zero mean Gaussian process with covariance Q and R, respectively; $\sigma_v = 5m/s$ is the standard deviation of the velocity process noise; $\sigma_w = 10m$ is the standard deviation of the measurement noise. The target number varies from 1 to 50. Targets move at constant speeds uniformly between 30 and 150 unit lengths per unit time so $\bar{v} = 150$. Targets appear from $J = 24$ possible locations or can be born at any time in these J possible locations with intensity

$$\gamma_t(x) = \sum_{i=1}^{J} \frac{1}{J} \mathcal{N}(x; m_\gamma^{(i)}, P_\gamma)$$

where $P_\gamma = diag(Pu_m^2)$, $P = [100, 100, 25, 25]$ and $u_m^2 = u_m^T u_m$, $u_m = \left[m, m, \frac{m}{s}, \frac{m}{s}\right]$ are used to model spontaneous births in the vicinity of $m_\gamma^{(i)}, i = 1, \ldots, J$. For illustration, target spawning is not considered. The ground truth is plotted with the presence of false alarms in figure 2.

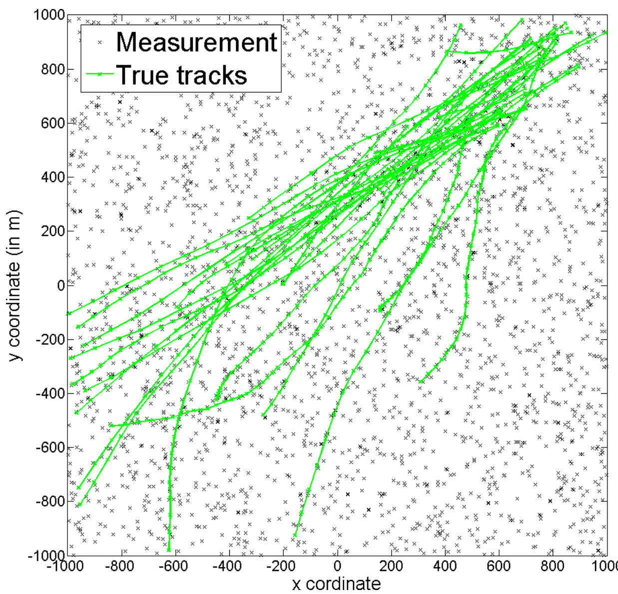

Fig. 2: Ground truth are plotted along with noisy measurements.

Each target survives with probability $P_S = 0.99$ and is detected with probability $P_D = 0.8$ so the maximum number of consecutive missed detection at any track is chosen as $\bar{d} = 2$. The detected measurements are immersed in clutter that is modeled as a Poisson RFS Λ_t with intensity

$$\lambda_c = \kappa_t V u$$

where u is the uniform density over the surveillance region, $V = 4 \times 10^6 m^2$, $\kappa_t = 12.5 \times 10^{(-6)} m^{-2}$ is intensity function and λ_c is the average number of clutter returns per unit volume (i.e. $\lambda_c = 50$ clutter returns per scan over $\mathcal{R}$).

A. Numerical result and Discussion

The problem of closely spaced and crossing targets cannot be solved reliably by popular filtering techniques with track gate $m^* = 3$. Our algorithm, PMMH for multi-target tracking, is designed to deal with this problem. After 141 accepted times out of 17079 runs, the estimates of the tracks from our algorithm are plotted against the ground truth in figure 3. The

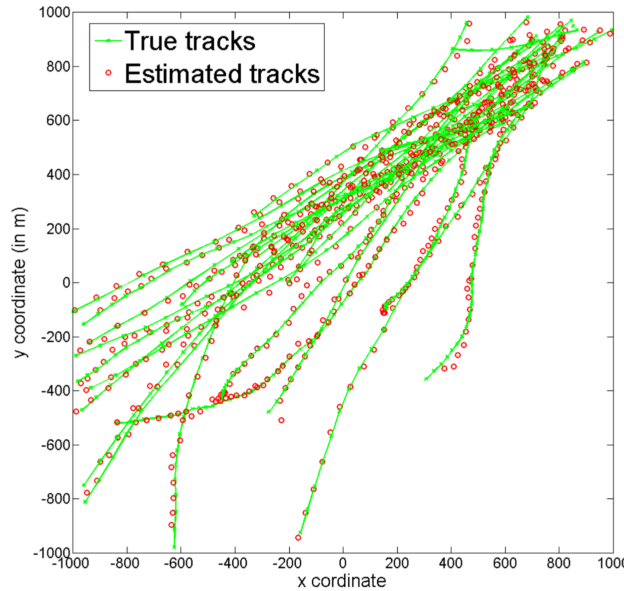

Fig. 3: The true tracks and estimated tracks from PMMH for MTT a with GMPHD output as initial state of a Markov chain.

performance of PMMH for MTT is evaluated through OSPA in figure 4. In this figure, there are some large errors which occurred at six different time scan periods such as at time scans $t = 1$, 5, 39, time intervals $9-10$, $41-42$ and $47-50$. Figures 5 and 7 explain the origin of these errors. These errors

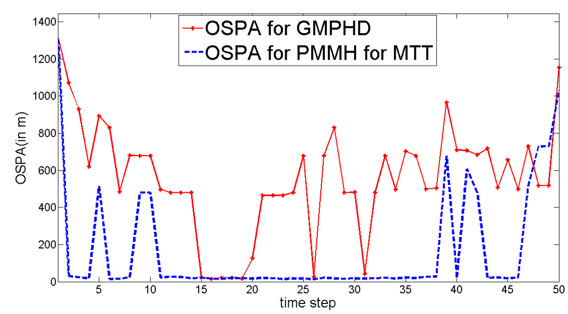

Fig. 4: The error between initial estimates from a GMPHD filter shown versus estimates from PMMH for MTT.

result from the miss-detection of the targets when targets first appear or before the targets disappear from the surveillance area. Figure 6 shows that the targets in which their states were not tracked by our algorithm are labeled and their trajectories are drawn in dashed line with cyan color. For examples, at time $t = 1$ the targets (labeled) 3 or 4 are born but not detected. This is similar to the target 10 at time $t = 5$. At time $t = 9$ and $t = 10$, the sensor does not detect the target 4 before the target disappears. This is also the case for the targets 16 and 17 at time $t = 39$. At time $t = 48, 49$ the target 30 is undetected and again is detected at the last scan T. There is not enough

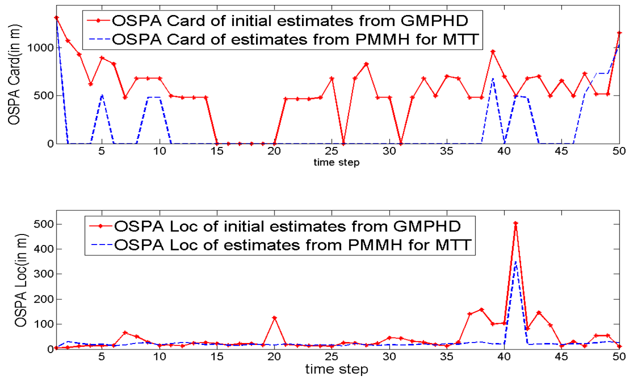

Fig. 5: Multi-target estimation (cardinality and localization) errors versus time for GMPHD and the PMMH for MMT.

information for our algorithm to confirm the state as the state of the target 30. The OSPA Loc also shows that whenever targets are detected during their existence period the location error seem to be small. However, the OSPA Card shows that there is an error during the time period between 47 and 50. This is because the number of targets during this time period is not correct. This happens because the target 33 only exists from time $t = 47$ to T but is only detected at every second time instant since $t = 48$. Therefore the PMMH for MTT algorithm does not have enough information to distinguish this target from a false target. This results in the loss of a track in the output of the PMMH for MTT algorithm.

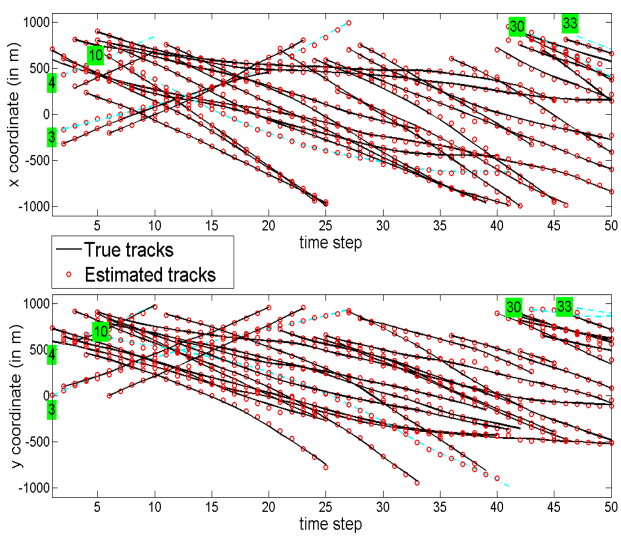

Fig. 6: Ground truth and its estimates are plotted versus time in which some states of labeled and cyan colored targets were not detected by PMMH for MTT algorithm.

B. Conclusion

A batch formulation and solution based on random finite sets for the MTT problem in a cluttered environment with low detection probabilities has been proposed in this paper. A simulation was successfully carried out on a moderately difficult scenario with medium probability detection ($P_D = 0.8$).

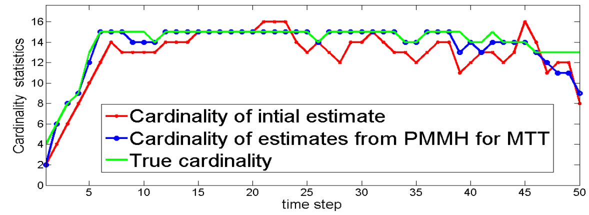

Fig. 7: True cardinality (green line) shown versus estimated cardinality of initial input (red line)- GMPHD filter, and PMMH for MTT (blue line).

The trajectories of a variable number of targets were tracked successfully. Tracking performance was reliable compared to standard filtering based MTT methods. However, the run-time cost is high for the batch method.

More difficult scenarios are currently under consideration.

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