Multi-Bernoulli Filtering with Unknown Clutter Intensity and Sensor Field-of-View

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***Abstract*—In Bayesian multi-target filtering knowledge of parameters such as clutter intensity and sensor field-of-view are of critical importance. Significant mismatches in clutter and sensor field of view model parameters results in biased estimates. In this paper we propose a multi-target filtering solution that can accommodate non-linear target model and unknown non-homogeneous clutter intensity and sensor field-of-view. Our solution is based on the multi-target multi-Bernoulli filter that adaptively learns non-homogeneous clutter intensity and sensor field-of-view while filtering.**

***Index Terms*—Multi-Target Bayes filter, multi-Bernoulli filter, multi-target tracking, online parameter estimation, robust filtering, finite set statistics.**

I. Introduction

Multi-target filtering, involves the joint estimation of the number of targets and their individual states from a sequence of observations. In addition to the non-linearity, process and measurement noise, the two main sources of uncertainty that pose significant challenges in multi-target filtering are clutter and detection, [1], [2], [8]. Clutter are spurious measurements that do not belong to any target. Detection uncertainty refers to the phenomena that the sensor does not always detect the targets due to non-ideal sensor field of view (FoV).

The random finite set (RFS) approach [8] is an elegant Bayesian formulation of multi-target filtering, which has resulted in the development of the Probability Hypothesis Density (PHD) filter [6] and the Cardinalized PHD (CPHD) filter [7]. Efficient implementations of these filters [15], [16], [17] have been gaining popularity in recent years. Convergence results have also been established by [3], [4]. A lesser known RFS-based multi-target filter is the (multi-target) multi-Bernoulli filter, which propagates the parameters of a multi-Bernoulli RFS that approximates the posterior multi-target RFS. For applications with highly non-linear models and/or non-homogeneous sensor FoV, the multi-Bernoulli filter offers an attractive alternative [18]. Multi-Bernoulli filtering with image observations has also been demonstrated in [5], [19]. Additionally, formal metrics for evaluating the performance of multi-target filters have also been proposed [13].

In Bayesian multi-target filtering, knowledge of parameters such as clutter rate and sensor field of view are of critical importance, arguably, more so than measurement noise model in single-target filtering. Significant mismatches in clutter and sensor FoV parameters inevitably result in erroneous estimates. However, except for some applications such as radar, sonar, and computer vision, clutter and FoV model parameters are not available in general. Moreover, these parameters depend on the detection method and it is not known whether they are time-invariant.

This paper proposes a solution to the non-linear multi-target filtering problem with uncertainty in clutter and sensor FoV models using the multi-Bernoulli filter. We show that the multi-Bernoulli filter can be used to adaptively learn non-homogeneous clutter intensity and FoV while filtering. A particle implementation of the multi-Bernoulli filter is used since it does not require assumptions such as linear Gaussian state space model nor homogeneous clutter intensity and FoV. Numerical results are provided to verify our approach.

Within the RFS framework, previous approaches to filtering with unknown clutter and detection parameters have focussed on moment based approximations via the CPHD and PHD filters [9], [10]. The work in [11] proposes CPHD filters for dealing with unknown clutter rate and/or detection profile, along with closed form implementations via Beta-Gaussian mixtures applicable to linear Gaussian target dynamics, as well as approximations to cope with mild non-linearities via extended and unscented Kalman techniques. Other related works such as [14] attempt to estimate the entire suite of multi-target model parameters with PHD filter using approximate gradient descent methods. To date however, there have been no works focussing on direct density based approximations to the full multi-target Bayes filter which cater directly for non-linear target models. This paper is the first to propose such a solution via multi-Bernoulli parameterized approximations.

II. The Multi-Bernoulli Filter

Suppose that at time k, there are $N(k)$ target states $\breve{x}_{k,1},\ldots,\breve{x}_{k,N(k)}$, each taking values in a state space $\breve{\mathcal{X}}$, and $M(k)$ observations $z_{k,1},\ldots,z_{k,M(k)}$ each taking values in an observation space $\mathcal{Z}$. Then, the multi-target state and multi-target observations, at time k, are the finite sets [6], [8], [18]

$$\breve{X}_k = \{\breve{x}_{k,1},\ldots,\breve{x}_{k,N(k)}\} \subset \breve{\mathcal{X}},$$
$$Z_k = \{z_{k,1},\ldots,z_{k,M(k)}\} \subset \mathcal{Z}.$$

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The multi-target Bayes filter is the fundamental mechanism for propagating the multi-target posterior density recursively in time. The stochastic variability of the multi-target state and multi-target observation are captured by modelling with RFSs. In this paper, the parameterized classes of Bernoulli and multi-Bernoulli RFSs are the focus of this modelling. Their basic definitions and probability densities are stated as follows. A *Bernoulli* RFS $\breve{X}$ on $\breve{\mathcal{X}}$ has probability $1-r$ of being empty, and probability r of being a singleton whose only element is distributed according to a probability density p defined on $\breve{\mathcal{X}}$. The mean cardinality of a Bernoulli RFS is r. The corresponding probability density is

$$\pi(\breve{X}) = \begin{cases} 1-r, & \breve{X}=\emptyset \\ rp(\breve{x}), & \breve{X}=\{\breve{x}\} \\ 0, & \textit{otherwise} \end{cases}$$

A Bernoulli RFS is completely described by the parameter pair (r,p). A *multi-Bernoulli* RFS $\breve{X}$ on $\breve{\mathcal{X}}$ is a union of a fixed number of independent Bernoulli RFSs $\breve{X}^{(i)}$ with existence probability $r^{(i)} \in (0,1)$ and probability density $p^{(i)}$ defined on $\breve{\mathcal{X}}$ for $i=1,...,M$, i.e.

$$X = \bigcup_{i=1}^{M} X^{(i)}. \tag{1}$$

The mean cardinality of a multi-Bernoulli RFS is $\sum_{i=1}^{M} r^{(i)}$. The corresponding probability density is

$$\pi(\{\breve{x}_1,...,\breve{x}_n\}) = \pi(\emptyset) \sum_{1\le i_1 \ne \cdots \ne i_n \le M} \prod_{j=1}^{n} \frac{r^{(i_j)} p^{(i_j)}(\breve{x}_j)}{1-r^{(i_j)}},$$

where $\pi(\emptyset) = \prod_{j=1}^{M}\left(1-r^{(j)}\right)$. A multi-Bernoulli RFS is completely described by the parameter set $\{(r^{(i)},p^{(i)})\}_{i=1}^{M}$.

The multi-Bernoulli filter [18] propagates the posterior density of the multi-target state as a multi-Bernoulli density recursively in time given a sequence of multi-target observations. The multi-Bernoulli filter is consequently a parameterized approximation to the multi-target Bayes filter [18]. The filter has the intuitive interpretation of propagating a time varying number of target tracks, each of which is described by a probability of existence as well as a probability density on the unknown state variable.

The prediction and update steps of the multi-Bernoulli filter [18] are summarized in the Propositions directly below. The following notation is used for the specification of the standard single target model which describes the transition, survival/death, and birth process of individual targets, as well as the measurement likelihood, detection/missed-detection and Poisson clutter or false alarm process

$$\begin{aligned}
p_{S,k}(\breve{\zeta}) &= \textit{probability of survival to time } k, \\
&\quad \textit{given previous state } \breve{\zeta}, \\
p_{D,k}(\breve{x}) &= \textit{probability of detection at time } k, \\
&\quad \textit{for current state } \breve{x}, \\
f_{k|k-1}(\cdot|\breve{\zeta}) &= \textit{single target transition to time } k, \\
&\quad \textit{given previous state } \breve{\zeta}, \\
g_k(\cdot|\breve{x}) &= \textit{single target likelihood at time } k, \\
&\quad \textit{given current state } \breve{x}, \\
\{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}} &= \textit{parameters of the multi-Bernoulli} \\
&\quad \textit{RFS of births at time } k, \\
\kappa_k(\cdot) &= \textit{intensity function of} \\
&\quad \textit{Poisson clutter at time } k.
\end{aligned}$$

Proposition: (Standard Prediction) If at time $k-1$, the posterior multi-target density is multi-Bernoulli of the form

$$\pi_{k-1} = \{(r_{k-1}^{(i)}, p_{k-1}^{(i)})\}_{i=1}^{M_{k-1}}, \tag{2}$$

then the predicted multi-target density is multi-Bernoulli given by

$$\pi_{k|k-1} = \{(r_{P,k|k-1}^{(i)}, p_{P,k|k-1}^{(i)})\}_{i=1}^{M_{k-1}} \cup \{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)})\}_{i=1}^{M_{\Gamma,k}}, \tag{3}$$

where

$$r_{P,k|k-1}^{(i)} = r_{k-1}^{(i)} \langle p_{k-1}^{(i)}, p_{S,k}\rangle, \tag{4}$$

$$p_{P,k|k-1}^{(i)}(\breve{x}) = \frac{\langle f_{k|k-1}(\breve{x}|\cdot), p_{k-1}^{(i)} p_{S,k}\rangle}{\langle p_{k-1}^{(i)}, p_{S,k}\rangle}. \tag{5}$$

Proposition: (Standard Update) If at time k, the predicted multi-target density is multi-Bernoulli of the form

$$\pi_{k|k-1} = \{(r_{k|k-1}^{(i)}, p_{k|k-1}^{(i)})\}_{i=1}^{M_{k|k-1}},$$

then, given a measurement set Z_k, the posterior multi-target density is multi-Bernoulli given by

$$\pi_k \approx \{(r_{L,k}^{(i)}, p_{L,k}^{(i)})\}_{i=1}^{M_{k|k-1}} \cup \{(r_{U,k}(z), p_{U,k}(\cdot;z))\}_{z\in Z_k}, \tag{6}$$

where

$$r_{L,k}^{(i)} = r_{k|k-1}^{(i)} \frac{1-\langle p_{k|k-1}^{(i)}, p_{D,k}\rangle}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)}, p_{D,k}\rangle}, \tag{7}$$

$$p_{L,k}^{(i)}(\breve{x}) = p_{k|k-1}^{(i)}(x)\frac{1-p_{D,k}(\breve{x})}{1-\langle p_{k|k-1}^{(i)}, p_{D,k}\rangle}, \tag{8}$$

$$r_{U,k}(z) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}(1-r_{k|k-1}^{(i)})\langle p_{k|k-1}^{(i)}, g_k(z|\cdot)p_{D,k}\rangle}{(1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)}, p_{D,k}\rangle)^2}}{\kappa_k(z) + \sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)}, g_k(z|\cdot)p_{D,k}\rangle}{1-r_{k|k-1}^{(i)}\langle p_{k|k-1}^{(i)}, p_{D,k}\rangle}}, \tag{9}$$

$$p_{U,k}(\breve{x};z) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1-r_{k|k-1}^{(i)}} p_{k|k-1}^{(i)}(\breve{x}) g_k(z|\breve{x}) p_{D,k}(\breve{x})}{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1-r_{k|k-1}^{(i)}} \langle p_{k|k-1}^{(i)}, g_k(z|\cdot)p_{D,k}\rangle}. \tag{10}$$

Estimation of the multi-target state also has an intuitive interpretation and proceeds by estimating the number of targets and then the individual target states. The posterior target cardinality can be estimated with either an expected a posteriori (EAP) or maximum a posteriori (MAP) estimate on the posterior cardinality distribution. Once established, the individual target states can be extracted by selecting the relevant number of posterior Bernoulli components with the highest existence probabilities, and computing either the EAP or MAP estimate for the corresponding posterior probability densities on the target state variable. A simple alternative is to select the Bernoulli components, whose existence probabilities exceed a predefined threshold, treat them as confirmed target tracks, and then extract individual state estimates from their corresponding state densities.

III. Unknown Clutter Intensity and Sensor FoV

Adapting the multi-Bernoulli filter to accommodate an unknown and non-homogeneous, clutter intensity and sensor FoV, can be achieved by deriving a particular version of the filter which is defined on a specially chosen state space. The approach presented here is adapted from that originally proposed for moment approximations such as the CPHD/PHD filters in [9], [10], and is adapted here specifically for parameterized density approximations such as the multi-Bernoulli filter. The underlying idea is explained as follows.

An unknown and state dependent sensor FoV is accommodated by incorporating an unknown detection probability into the target state variable. This is achieved by considering an augmented state given by the usual kinematic state as well as an augmented variable which represents a corresponding unknown detection probability. The intuition here is that the filter should implicitly estimate the unknown detection probability as a matter of course.

An unknown and non-homogeneous clutter intensity is accommodated by dropping the standard Poisson assumption for false alarms, and modelling individual clutter returns based on individual clutter targets/objects or generators. Each clutter generator is analogous to an actual target/object, in the sense that clutter generators have their own separate models for births and deaths as well as transition, likelihood and detection or missed detection. However, the two types of clutter and actual targets/objects are distinct, and cannot evolve into the other type. The intuition here is that the clutter generators will dynamically distribute themselves around the state space to explain the prevailing false alarm conditions.

Formally, we proceed as follows. Let $\mathcal{X}^{(\Delta)}=[0,1]$ denote the space of detection probabilities, $\mathcal{X}=\mathbb{R}^{n_x}$ denote the space for the target kinematics, and $\{0,1\}$ denote the discrete space of labels for clutter generators and actual targets. The convention that a label of $u=0$ denotes a clutter generator and a label of $u=1$ denotes an actual target will be used throughout. The new state space is given by

$$\breve{\mathcal{X}}=\mathcal{X}^{(\Delta)}\times\mathcal{X}\times\{0,1\},$$

where $\times$ denotes a Cartesian product. Consequently, the state variable takes on the value $\breve{x}=(a,x,u)=(augmentation, kinematics, label)$. The following convention regarding the discrete target label and arbitrary functions defined on the new state space is used throughout

$$f(a,x,u)=f^{(u)}(a,x).$$

Integration on the new state space is given by

$$\int_{\breve{\mathcal{X}}} f(\breve{x})d\breve{x}=\int_{\mathcal{X}^{(\Delta)}\times\mathcal{X}} f^{(0)}(a,x)dadx+\int_{\mathcal{X}^{(\Delta)}\times\mathcal{X}} f^{(1)}(a,x)dadx.$$

In the following, an explicit specification of the single target models on the new state space is given. Note that clutter generators and actual targets have separate model parameters, and specifically that, clutter generators can never become actual targets and vice-versa. Note also that the standard Poisson clutter term has accordingly been set to 'zero' with $\kappa_k(z)\equiv 0$, since false alarms are now modelled by dynamic clutter generators. For the specification block below, the single-target models for actual targets and clutter generators are parameterized by the variable u. Setting $u=1$ yields the complete single target model for actual targets, while substitution of $u=0$ yields the complete single target model for clutter generators:

$$\begin{aligned}
p_{S,k}(\alpha,\zeta,u) &= p_{S,k}^{(u)}(\zeta)\\
&= \textit{probability of survival to time } k,\\
&\quad\ \textit{given previous state } \zeta,\\
p_{D,k}(a,x,u) &= p_{D,k}^{(u)}(a,x)=a\\
&= \textit{probability of detection at time } k,\\
&\quad\ \textit{for kinematic state } x,\\
f_{k|k-1}(a,x,u|\alpha,\zeta,u) &= f_{k|k-1}^{(u)}(a,x|\alpha,\zeta)\\
&= f_{k|k-1}^{(u)(\Delta)}(a|\alpha)f_{k|k-1}^{(u)(\mathcal{X})}(x|\zeta)\\
&= \textit{transition density to time } k,\\
&\quad\ \textit{for target state,}\\
&\quad\ \textit{given previous state } \alpha,\zeta,\\
f_{k|k-1}^{(u)(\Delta)}(a|\alpha) &= \textit{transition density to time } k,\\
&\quad\ \textit{for detection probability,}\\
&\quad\ \textit{given previous value } \alpha,\\
f_{k|k-1}^{(u)(\mathcal{X})}(x|\zeta) &= \textit{transition density to time } k,\\
&\quad\ \textit{for kinematic state,}\\
&\quad\ \textit{given previous value } \zeta,\\
g_k(z|a,x,u) &= g_k^{(u)}(z|x)\\
&= \textit{likelihood at time } k,\\
&\quad\ \textit{given current state } x,\\
\{(r_{\Gamma,k}^{(i)(u)},p_{\Gamma,k}^{(i)(u)})\}_{i=1}^{M_{\Gamma,k}^{(u)}} &= \textit{parameters of the multi-Bernoulli}\\
&\quad\ \textit{RFS of births at time } k.
\end{aligned}$$

The following propositions establish the prediction and update steps of the proposed multi-Bernoulli filter. These follow as a direct consequence of applying the new state space and single target models to the standard multi-Bernoulli filter. The proof is tedious but routine and is consequently omitted.

A. Recursion

Proposition: (Prediction) If at time $k-1$, the posterior multi-target density is multi-Bernoulli of the form

$$\pi_{k-1}=\{(r_{k-1}^{(i)},p_{k-1}^{(i)})\}_{i=1}^{M_{k-1}},$$

where $p_{k-1}^{(i)}(\cdot,\cdot,u)=p_{k-1}^{(i)(u)}(\cdot,\cdot)$ for $u=0,1$, then the predicted multi-target density is multi-Bernoulli given by

$$\pi_{k|k-1}=\bigcup_{u=0,1}\{(r_{\Gamma,k}^{(i)(u)},p_{\Gamma,k}^{(i)(u)})\}_{i=1}^{M_{\Gamma,k}^{(u)}}\cup\{(r_{k|k-1}^{(i)},p_{P,k|k-1}^{(i)})\}_{i=1}^{M_{k-1}},\tag{11}$$

where

$$r_{P,k|k-1}^{(i)}=r_{k-1}^{(i)}\sum_{u=0,1}\langle p_{k-1}^{(i)(u)},p_{S,k}^{(u)}\rangle,\tag{12}$$

$$p_{P,k|k-1}^{(i)(u)}(a,x)=\frac{\langle f_{k|k-1}^{(u)}(a,x|\cdot,\cdot),p_{k-1}^{(i)(u)}p_{S,k}^{(u)}\rangle}{\langle p_{k-1}^{(i)(u)},p_{S,k}^{(u)}\rangle}.\tag{13}$$

Proposition: (Update) If at time k, the predicted multi-target density is multi-Bernoulli of the form

$$\pi_{k|k-1} = \{(r^{(i)}_{k|k-1}, p^{(i)}_{k|k-1})\}_{i=1}^{M_{k|k-1}},$$

where $p^{(i)}_{k|k-1}(\cdot,\cdot,u) = p^{(i)(u)}_{k|k-1}(\cdot,\cdot)$ for $u = 0,1$, then given a measurement set Z_k, the posterior multi-target density is multi-Bernoulli given by

$$\pi_k = \{(r^{(i)}_{L,k}, p^{(i)}_{L,k})\}_{i=1}^{M_{k|k-1}} \cup \{(r_{U,k}(z), p_{U,k}(\cdot;z))\}_{z\in Z_k}, \tag{14}$$

where for $u = 0,1$

$$r^{(i)}_{L,k} = \sum_{u=0,1} r^{(i)(u)}_{L,k}, \tag{15}$$

$$r^{(i)(u)}_{L,k} = \frac{r^{(i)}_{k|k-1}\langle p^{(i)(u)}_{k|k-1}, 1-p^{(u)}_{D,k}\rangle}{1 - r^{(i)}_{k|k-1}\sum_{\ell=0,1}\langle p^{(i)(\ell)}_{k|k-1}, p^{(\ell)}_{D,k}\rangle}, \tag{16}$$

$$p^{(i)(u)}_{L,k}(a,x) = \frac{(1-a)p^{(i)(u)}_{k|k-1}(a,x)}{\sum_{\ell=0,1}\langle p^{(i)(\ell)}_{k|k-1}, 1-p^{(\ell)}_{D,k}\rangle}, \tag{17}$$

$$r_{U,k}(z) = \sum_{u=0,1} r^{(u)}_{U,k}(z), \tag{18}$$

$$r^{(u)}_{U,k}(z) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r^{(i)}_{k|k-1}(1-r^{(i)}_{k|k-1})\langle p^{(i)(u)}_{k|k-1}, g^{(u)}_k(z|\cdot)p^{(u)}_{D,k}\rangle}{(1-r^{(i)}_{k|k-1}\sum_{\ell=0,1}\langle p^{(i)(\ell)}_{k|k-1}, p^{(\ell)}_{D,k}\rangle)^2}}{\sum_{i=1}^{M_{k|k-1}} \frac{r^{(i)}_{k|k-1}\sum_{\ell=0,1}\langle p^{(i)(\ell)}_{k|k-1}, g^{(\ell)}_k(z|\cdot)p^{(\ell)}_{D,k}\rangle}{1-r^{(i)}_{k|k-1}\sum_{\ell=0,1}\langle p^{(i)(\ell)}_{k|k-1}, p^{(\ell)}_{D,k}\rangle}} \tag{19}$$

$$p^{(u)}_{U,k}(a,x;z) = \frac{\sum_{i=1}^{M_{k|k-1}} \frac{r^{(i)}_{k|k-1}}{1-r^{(i)}_{k|k-1}} p^{(i)(u)}_{k|k-1}(a,x) g^{(u)}_k(z|x)\cdot a}{\sum_{\ell=0,1}\sum_{i=1}^{M_{k|k-1}} \frac{r^{(i)}_{k|k-1}}{1-r^{(i)}_{k|k-1}} \langle p^{(i)(\ell)}_{k|k-1}, g^{(\ell)}_k(z|\cdot)p^{(\ell)}_{D,k}\rangle} \tag{20}$$

B. State Estimation

Estimation of the multi-target state proceeds in the usual manner by first estimating the number of actual targets and subsequently the individual target states. However, since the target state describes both actual targets and clutter generators, the standard state estimation routine previously described is not directly applicable. The number of targets must be estimated as an EAP estimate by considering only the parts of the existence probabilities which pertain to actual targets, i.e. $\hat{N}_k = \sum_{i=1}^{M_k} r^{(i)(1)}_k$. Note that a MAP estimate on the posterior cardinality distribution is not appropriate, since the posterior cardinality distribution implicitly computed by the filter includes both actual targets and clutter generators. An estimate of the variance on the target number estimate is $\hat{\sigma}^2_{N,k} = \sum_{i=1}^{M_k} r^{(i)(1)}_k(1 - r^{(i)(1)}_k)$. Extraction of individual target states must then be computed as either an EAP or MAP estimator on the relevant Bernoulli components, by considering the corresponding probability density $p^{(\cdot)(1)}_k(a,x)$, where the augmented parameter can be treated as a nuisance variable and integrated out, or reported as an estimate of the detection probability a at the location x. The mean rate of clutter can be simply estimated as $\hat{\lambda}_{c,k} = \sum_{i=1}^{M_k} r^{(i)(0)}_k \int a p^{(i)(0)}_k(a,x)dadx$.

IV. Demonstration

A non-linear multi-target tracking scenario is used to demonstrate the performance of the proposed multi-Bernoulli filter. Implementation is via particle or Sequential Monte Carlo methods, adapted directly from those already proposed for the standard multi-Bernoulli filter. A total of 10 targets appear on the scene throughout the scenario. Target tracks are shown in Figure 1 on the half disc of radius $2000m$ with the start and stop positions of each track. The target state $\breve{x}_k = [\ a_k, p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}, \omega_k, u_k\ ]^T$ comprises the the unknown detection probability a_k, the planar position and velocity $\ddot{x}_k = [\ p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}\ ]^T$, the turn rate ω_k, and the type label $u_k \in \{0,1\}$. Sensor returns are bearings and range vectors of the form $z_k = [\ \theta_k, r_k\ ]^T$. The model parameters are given below where the notation $\mathcal{N}(\cdot;m,P)$ is used to denote a Gaussian density with mean and covariance m,P and the notation $\beta(\cdot;s,t)$ is used to denote a Beta density with parameters s,t.

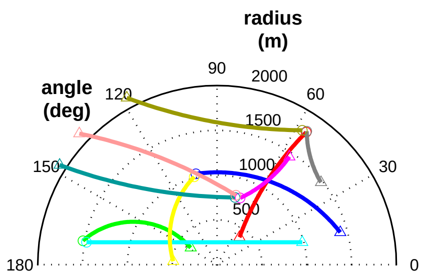

Fig. 1. Target trajectories in polar coordinates. Start/Stop positions for each track are shown with ○/△.

Model for Actual Targets: Actual targets follow a co-ordinated turn model on the position, velocity and turn rate variables $x_k = [\ddot{x}_k, \omega_k]$, with transition density given by

$$f^{(1)(\mathcal{X})}_{k|k-1}(x_k|x_{k-1}) = \mathcal{N}(x_k; m^{(1)}_{x,k|k-1}(x_{k-1}), P^{(1)}_{x,k|k-1}),$$

where $m^{(1)}_{x,k|k-1}(x_{k-1}) = [F(\omega_{k-1})\ddot{x}_{k-1}, \omega_{k-1}]^T$, $P^{(1)}_{x,k|k-1} = \mathrm{diag}([\sigma^2_w GG^T, \sigma^2_u])$,

$$F(\omega) = \begin{bmatrix} 1 & \frac{\sin\omega\Delta}{\omega} & 0 & -\frac{1-\cos\omega\Delta}{\omega} \\ 0 & \cos\omega\Delta & 0 & -\sin\omega\Delta \\ 0 & \frac{1-\cos\omega\Delta}{\omega} & 1 & \frac{\sin\omega\Delta}{\omega} \\ 0 & \sin\omega\Delta & 0 & \cos\omega\Delta \end{bmatrix}, G = \begin{bmatrix} \frac{\Delta^2}{2} & 0 \\ T & 0 \\ 0 & \frac{\Delta^2}{2} \\ 0 & \Delta \end{bmatrix},$$

and $\Delta = 1s$ is the sampling time, $\sigma_w = 15m/s^2$ is the standard deviation of the process noise, $\sigma_u = 3\pi/180rad/s$ is the standard deviation of the turn rate noise. The transition density for the augmented part of the state is

$$f^{(1)(\cdot)}_{k|k-1}(a_k|a_{k-1}) = \beta(a_k; s^{(1)}_{k|k-1}, t^{(1)}_{k|k-1}),$$

where

$$s^{(1)}_{k|k-1} = \left(\mu^{(1)}_{a,k|k-1}(1-\mu^{(1)}_{a,k|k-1})/[\sigma^{(1)}_{a,k|k-1}]^2 - 1\right)\mu^{(1)}_{a,k|k-1},$$

$$t^{(1)}_{k|k-1} = \left(\mu^{(1)}_{a,k|k-1}(1-\mu^{(1)}_{a,k|k-1})/[\sigma^{(1)}_{a,k|k-1}]^2 - 1\right)(1-\mu^{(1)}_{a,k|k-1}),$$

gives a chosen (matching) mean of $\mu^{(1)}_{a,k|k-1} = a_{k-1}$ and a chosen (a priori) standard deviation of $\sigma^{(1)}_{a,k|k-1} = 0.01$ for this transition. The label variable is fixed at $u_k \equiv 1$ for actual targets. The survival probability for actual targets is $p_{S,k}^{(1)}(x_k) = 0.99$. Actual targets produce noisy bearings and range measurements $z_k = [\ \theta_k, r_k\ ]^T$ with likelihood given by

$$g_k^{(1)}(z_k|x_k) = \mathcal{N}(z_k; m_{z,k}^{(1)}(x_k), P_{z,k}^{(1)}),$$

where

$$\begin{aligned} m_{z,k}^{(1)} &= [\arctan(p_{x,k}/p_{y,k}), \sqrt{p_{x,k}^2 + p_{y,k}^2}], \\ P_{z,k} &= \text{diag}([\ \sigma_\theta^2, \sigma_r^2\ ]^T), \end{aligned}$$

$\sigma_\theta = (\pi/180)rad$ and $\sigma_r = 5m$. Note that the probability of detection for actual targets is unknown to the filter, but the measurement data is simulated according to a state dependent detection probability, which peaks at value of $p_{D,k} = 0.98$ at the origin and tapers off to a value of $p_{D,k} = 0.92$ at the edge of the surveillance region. The actual target birth process has probability density $\pi_{\Gamma,k}^{(1)} = \{(r_{\Gamma,k}^{(i)(1)}, p_{\Gamma,k}^{(i)(1)})\}_{i=1}^4$ where $r_{\Gamma,k}^{(1)(1)} = r_{\Gamma,k}^{(2)(1)} = 0.02$, $r_{\Gamma,k}^{(3)(1)} = r_{\Gamma,k}^{(4)(1)} = 0.03$, $p_{\Gamma,k}^{(i)(1)}(a_k, x_k) = \beta(a; s_{\Gamma,k}^{(i)(1)}, t_{\Gamma,k}^{(i)(1)})\mathcal{N}(x; m_{\Gamma,k}^{(i)(1)}, P_{\Gamma,k}^{(i)(1)})$, $s_{\Gamma,k}^{(i)(1)} = 98$, $t_{\Gamma,k}^{(i)(1)} = 2$, $m_{\Gamma,k}^{(1)(1)} = [\ -1500, 0, 250, 0, 0\ ]^T$, $m_{\Gamma,k}^{(2)(1)} = [\ -250, 0, 1000, 0, 0\ ]^T$, $m_{\Gamma,k}^{(3)(1)} = [\ 250, 0, 750, 0, 0\ ]^T$, $m_{\Gamma,k}^{(4)(1)} = [\ 1000, 0, 1500, 0, 0\ ]^T$, and $P_{\Gamma,k}^{(i)(1)} = \text{diag}([\ 90,\ 90,\ 90,\ 90,\ 9(\pi/180)\ ]^T)^2$.

Model for Clutter Generators: In this model, clutter generators are modelled only by their (unknown) detection probability a_k and planar positions $\ddot{x}_k = [\ p_{x,k}, p_{y,k}\ ]^T$, thus their velocities and turn rate are ignored. Clutter generators follow a random walk model on the positional components of the state, given by the transition density $f_{k|k-1}^{(0)(\mathcal{X})}(\ddot{x}_k|\ddot{x}_{k-1}) = \mathcal{N}(\ddot{x}_k; \ddot{x}_{k-1}, P_{x,k|k-1}^{(0)})$ where $P_{x,k|k-1}^{(0)} = \text{diag}([\sigma_x^2, \sigma_y^2])$ and $\sigma_x = 1000m$ and $\sigma_y = 500m$ are the noise standard deviations on each axis. The transition density for the augmented part of the state $f_{k|k-1}^{(0)(\cdot)}(a_k|a_{k-1})$ is given by a Beta density, similar to the expression for actual targets, except with a standard deviation of $\sigma_{a,k|k-1}^{(0)} = 0.07$. The label variable is fixed at $u_k \equiv 0$ for clutter generators. The survival probability for clutter generators is $p_{S,k}^{(0)}(\ddot{x}_k) = 0.90$. Clutter generators also produce noisy bearings and range measurements with likelihood $g_k^{(0)}(z_k|x_k)$, similar to the equation for actual targets, except with noise standard deviations $\sigma_\theta = 20(\pi/180)rad$ and $\sigma_r = 400m$. The probability of detection for clutter generators is similarly unknown to the filter. The clutter generator birth process has probability density $\pi_{\Gamma,k}^{(0)} = \{(r_{\Gamma,k}^{(i)(0)}, p_{\Gamma,k}^{(i)(0)})\}_{i=1}^{20}$ where $r_{\Gamma,k}^{(i)(0)} = 0.1$, $p_{\Gamma,k}^{(i)(0)}(a_k, \ddot{x}_k) = \beta(a_k; s_{\Gamma,k}^{(i)(0)}, t_{\Gamma,k}^{(i)(0)})\mathcal{U}(\ddot{x}_k)$ and $s_{\Gamma,k}^{(i)(0)} = t_{\Gamma,k}^{(i)(0)} = 5$, $\mathcal{U}(\cdot) = $ uniform. The number of clutter returns is simulated according to a Binomial distribution with 20 generators and probability 0.5 giving an average of 10 returns per scan, and the spatial distribution of clutter is simulated such that it is increasingly concentrated near the origin and increasingly sparse moving radially outwards.

Filter Parameters: The filter enforces a maximum of $L_{\max} = 1000$ and minimum of $L_{\min} = 100$ particles per Bernoulli component or track. The actual number of particles in each component or track is allocated proportional to its existence probability. For both actual targets and clutter generators, their respective transition densities are used as proposals in the prediction step. To generate new samples representing spontaneous births of actual targets, the birth density itself is used as the proposal since it can be easily sampled from directly. To generate new samples representing spontaneous births of clutter generators, a special measurement driven proposal is used based on the approach in [12], so as to minimize wastage of particles since the a priori birth density is uniform. Track pruning is performed with a threshold of $T_{\min} = 10^{-4}$. The filter is initialized with zero actual targets and entirely with uniformly distributed clutter generators.

Results: The filter output for a single run is shown in Figure 2 giving the x and y coordinates of the true and estimated positions, along with the x and y coordinates of the received measurements versus time. It can be seen that the filter has reasonable performance, generally initiating and terminating each of the tracks within a several time steps, and generally producing accurate estimates of the target positions. Occasional incidences of false and dropped tracks are encountered, although this is to be expected, since the clutter intensity and sensor FoV are not known and must be dynamically estimated.

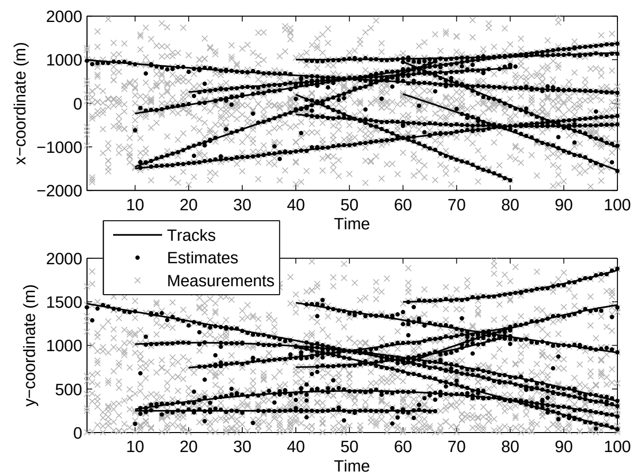

Fig. 2. Multi-Bernoulli filter for unknown non-homogenous clutter intensity and sensor FoV: estimates and tracks in x and y coordinates versus time

V. Conclusion

This paper has proposed a robust multi-Bernoulli filter along with implementations for the tracking of an unknown and time varying number of targets in the presence of noise, detection uncertainty, and false alarms. The filter is able to naturally accommodate non-linear targets models, and more importantly, accommodate an unknown and non-homogeneous clutter intensity and sensor field-of-view. The proposed algorithm is able to simultaneously and adaptively learn these unknown parameters in the process of performing multi-target filtering. Demonstrations show that the proposed filter performs acceptably in a tracking scenario with non-linear target dynamics and measurements. However, as expected, the proposed filter is more prone to false and dropped tracks, compared to the standard multi-Bernoulli filter which is supplied with and assumes the correct clutter and detection model.

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